

6/12/17

Fractions - Percents - Decimals H.W #1

Fractions \rightarrow decimals \rightarrow Percent

ex: $\frac{3}{4} = .75 = 75\%$
more decimal 2 places
 \rightarrow , add % sign or multi
by 100, add % sign

ex: $\frac{4}{7} = .571 = 57.1\%$
Round to 3 decimal places
 $.571 \cdot 100 = 57.1\%$

In a survey, 82% of 1463 people have cell phones.

- (a) What is 82% of 1463?
- (b) Could this be the actual #?
- (c) What is the actual #?

(a) $1463(82\%) = 1463(.82) = \boxed{1199.66}$

(b) No, can't have .66 of a person

(c) Probably 1200 actual #

ex: 33 of 115 people have cell phones.
what % is that? Round to 2 decimal places.

$$\frac{33}{115} = \underline{.2869565} = 28.69565\% = \boxed{28.70\%}$$

To round to 2 decimal places %, you need 5 decimal places

TRY: In a survey, 37% of 992 people have seen a UFO.

Ⓐ What is 37% of 992

Ⓑ What is the actual # who's seen a UFO

Ⓒ 992

$\underline{x \cdot 37} \leftarrow 37\%$

367.04 calculate

Ⓓ 367

TRY: If 211 of 763 have seen a UFO, what % is that, round to 3 decimal places.

$$211/763 = \underline{.27653} = 27.653\% = 27.65\% \text{ (2)}$$

Binomial Distribution H.W II

A random variable X that has the binomial distribution, has a mean, (expected value) $\mu = \text{mean}$ and a standard deviation $\sigma = \text{sigma}$

$$\mu = np = (\# \text{ of trials}) \cdot (\text{probability of success})$$

$$\sigma = \sqrt{npq} \quad q = 1 - p$$

Range of usual value (rouv)
 $= [\mu - 2\sigma, \mu + 2\sigma]$

ex: $X = \#$ of heads in 10 flips. Find μ, σ, rouv

$$\mu = np = 10(0.5) = 5$$

$$\sigma = \sqrt{npq} = \sqrt{10(0.5)(0.5)} = 1.5 \rightarrow \boxed{1.6}$$

$$\text{rouv} = [\mu - 2\sigma, \mu + 2\sigma]$$

$$= [5 - 2(1.6), 5 + 2(1.6)]$$

$$= [\boxed{1.8}, \boxed{8.2}]$$

H.W Help #10 Finish

Ex: An airplane that 94% of people who buy a ticket actually show up. If an airline sells 25 tickets with 23 seats, find the probability (not enough seats)

$$= P(\text{more than 23 successes})$$

$$= 1 - \text{binomcdf}(25, .94, 23) = .553$$

Ex: (acceptance sampling) In a large batch of aspirin $P(\text{good one}) = .999$.

A batch is accepted if among 6 aspirin, there is at most 1 bad. Find $P(\text{batch accepted})$

Bad one is success, so $P(\text{success}) = 1 - .999 = .001$
 Rule of complements:

$$P(\text{at most 1 success}) = \text{binomcdf}(6, .001, 1) = .999$$

TRY: A batch is accepted if among 15 tires there are at most 2 bad.

Find $P(\text{batch accepted})$

$$n=15, p=.007, x=2; (15, .007, 2) = .999$$

: binomcdf

① wrong: HW: ||

change to % move decimal
over!! net .99 / right 99.33

W/L:

6/12/17

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Descriptive Statistics, Ch 3

3.2 Measure of Center

1. Mean
2. Median
3. Mode
4. Mean Range
5. Weighted Mean

ex. You got 78, 85, 88, and 93
on 4 tests, what was the mean?

Data Points: 78, 85, 88, 93
 x_1 x_2 x_3 x_4

4 data points

$$\bar{X} = \frac{\sum_{k=1}^n x_k}{n} = \frac{(78 + 85 + 88 + 93)}{4} = 86$$

↓
sample mean

ex: Maple leaf won 42, 24 and 37
games past 3 years, find the mean

$$\bar{X} = \frac{(42 + 24 + 37)}{3} = 34.33 = 34.3$$

Round off for \bar{X} : to one more decimal
Place than data points

Descriptive Statistics

median = If you make a sorted list low to high of the data, median is the data point in the middle of the list

ex: Find the median 16, 22, 14, 17, 43
 Sorted list 14, 16, 17, 22, 43
 median = 17

ex: Find median 22, 18, 57, 35
 Sorted list 18, 22, 35, 57

median = mean these 2 #'s

$$= \frac{(22 + 35)}{2} = 28.5$$

mode = Data point that occurs most often

ex: Find mode: (a) 2, 2, 2, 2 mode = 2

(b) 1, 2, 1, 2, 1, 2 bimodal, modes = 1, 2

(c) no repeat = no mode

midrange = mean of high and low \rightarrow

$$\frac{\quad + \quad}{2}$$

ex: Find the midrange for 6, 12, 25, 3, 14

$$\text{midrange} = \frac{25 + 3}{2} = 14$$

midrange
low | high

H.W.
#3

Weighted mean = mean in which data points are assigned "weights"

Example of weighted mean: GPA

Find the GPA

	Grade	units
Theory mes.	B = 3	4
Topology	A = 4	3
Diff. Geo	C = 2	5
Set Theory	B = 3	5

multiply Grade points units for each class
3, 4, 2, 3

Add all up, \div results by total # units
4, 3, 5, 5

$$\text{GPA} = \frac{(3 \cdot 4 + 4 \cdot 3 + 2 \cdot 5 + 3 \cdot 5)}{(4 + 3 + 5 + 5)} = 2.88$$

Find GPA

	Grade	units
math	B = 3	4
English	A = 4	5
Pol. Si	A = 4	3
Art	A = 4	4

$$\text{GPA! } \frac{(3 \cdot 4 + 4 \cdot 5 + 4 \cdot 3 + 4 \cdot 4)}{(4 + 5 + 3 + 4)} = 3.75 \dots$$

TRY: Find GPA

	Grade	Units
math	C = 2	5
English	B = 3	3
Pol. Si	B = 3	5
Pho	D = 1	3

$$\text{GPA! } \frac{(2 \cdot 5 + 3 \cdot 3 + 3 \cdot 5 + 1 \cdot 3)}{(5 + 3 + 5 + 3)} = \frac{37}{16} = 2.31$$

Measure of Variation

Range

Standard Deviation

Variance

"Usual Values"

Range is high - low

ex: Find range: 2, 16, 14, 22, 116

$$\text{range} = 116 - 2 = 114$$

$$\begin{array}{c} | \text{range} = | \\ 2 \quad 114 \quad 116 \end{array}$$

ex: Find \bar{X} \textcircled{a} 50, 50, 50, 50 = $\bar{X} = 50$ no variation

\textcircled{b} 1, 99, 2, 98 $\bar{X} = 50$ lots variation

Standard Deviation S is a measure of the variation in a data set

no variation what-so-ever, example \textcircled{a}
 $S = 0$

$$\text{Formula: } S = \sqrt{\frac{\sum (x_i - \bar{X})^2}{n-1}}$$

$X_i - \bar{X}$ = Distance from each data point to the mean of data set

÷ the sum of those distance by $n-1$, gives us, sort of, the mean distance of the data points to the mean of the data set

ex: Find Std. dev. of 27, 47, 16, 25, 33, 15

Calculate

STAT Button → clear list

Clr List

2nd L1 → Enter

Done

STAT

Edit Enter, enter list #'s Enter

STAT → **Calc** → 1 Var Enter

2nd → L1 → Enter

$\bar{X} = 26.3 = \text{mean}$

$S_x = S = 10.3 = \text{std. dev. (round 1 decimal)}$

$\sigma_x = \sigma = 9.4$ (not this one)

Population std. dev.

Inferential Statistics

Given sample data make an inference about the population

1. Confidence intervals
2. Hypothesis testing

ex: Kitchen Sink

Find mean - Std. dev - Variance - range of Usual values and identify any usual values of following data set

16, 18, 7, 33, 14, 11, 365

mean = $\bar{X} = 66.293$ ← Round
Std. dev = $S_x = 131.97$

$$\text{Variance} = S^2 = \frac{\sum (X_i - \bar{X})^2}{n-1}$$

Sq. All the digits of S and then round off for S^2

$$S^2 = \text{variance} = (131.9743842)^2 = \underline{17417.2}$$

Range of Usual Value is
 $(\bar{X} - 2s, \bar{X} + 2s)$

ALL the value within 2 standard deviation of the mean

unusual | usual | values | usual
 $\bar{X} - 2s$ \bar{X} $\bar{X} + 2s$

ex: $\bar{X} = 66.3$
 $S = 132$

$(\bar{X} - 2s, \bar{X} + 2s)$

$= (66.3 - 2(132), 66.3 + 2(132))$
 $-197 \qquad 330.3$

= range of usual value

365 is Unusual

365 ← unusual



6-14-2015

measures of Variation H.W 3

1. Range
2. STD. dev.

Kit... Sink

ex: Find mean, std. dev, range of usual values, and identify any unusual values:

2, 41, 68, 69, 75, 84, 116

Calculate

STAT

Clear List \rightarrow Clr list

2nd \rightarrow L1 \rightarrow Enter \rightarrow Done

STAT \rightarrow Edit \rightarrow Enter

enter # List \rightarrow Enter

STAT \rightarrow Calc \rightarrow [1 Var]

Enter \rightarrow 2nd \rightarrow L1 \rightarrow Enter

$$\bar{X} = 65 \text{ range}$$

$$S = 35.7 \text{ std. dev.}$$

$$\text{ROUV} = [\bar{x} - 2s, \bar{x} + 2s]$$

$$= [65 - 2(35.7), 65 + 2(35.7)]$$

$$= [-6.4, 136.4] \text{ no unusual values}$$

TRY: Find \bar{X} , S , ROUV, Identify any unusual: 77, 65, 63, 45, 33, 189

$$\bar{X} = 75.3$$

$$S = 52.1$$

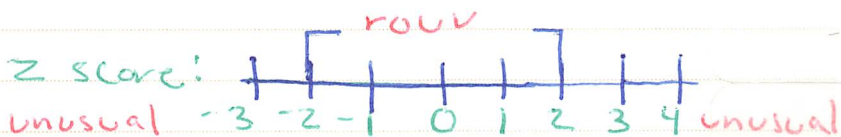
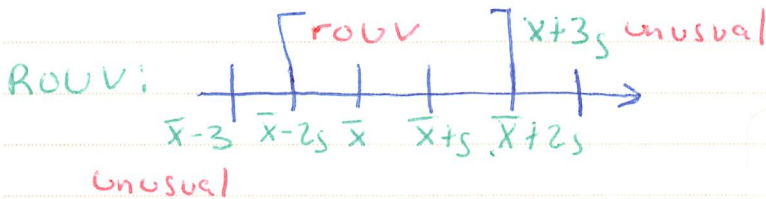
$$\text{ROUV} = [\bar{X} - 2s, \bar{X} + 2s]$$

$$= [75.3 - 2(52.1), 75.3 + 2(52.1)]$$

$$= [-28.9, 179.5] \text{ unusual } 189$$

Measure of Relative Standing H.W #4 Z-scores!

Are measuring data points in terms of units of standing deviations above or below the mean



Formula: $Z = \frac{X - \bar{X}}{S}$ mean
std. dev.

Ex: Find Z if $X = 14.7$, $\bar{X} = 14.3$, $S = 2.5$

Point Pop. Avg.

$$Z = \frac{(14.7 - 14.3)}{2.5} = .16 \text{ usual}$$

L std. dev. Round 2 decimal

Ex: Same Question

$$X = 3.8, \bar{X} = 7.2, S = .45$$

Point Pop. Avg

$$Z = \frac{X - \bar{X}}{S} = \frac{3.8 - 7.2}{.45} = -7.56$$

std. dev. unusual

Usual Z in $[-2, 2]$

unusual if $Z > 2$ or $Z < -2$

TRY: Find Z , usual or unusual?

$$\overset{****}{X} = 42.6, \bar{X} = 40.5$$

$S = 1.3$

$$Z = \frac{42.6 - 40.5}{1.3} = 1.62 \text{ usual}$$

A characterization of Z scores!

1. If $Z < 0$ (ie $Z = \text{neg}$)
data point, below mean
↓
2. If $Z > 0 \Rightarrow$ data point above mean
↑
3. If $Z = 0 \Rightarrow$ data point = to the mean

Probability HW # 5

1. Basic Definitions
2. Addition rule
3. multiplication rule
4. At least 1 principle
5. Conditional probability

When they say 20% chance of rain,
... they look at all situations in data base
that are similar to current situation,
and determine that 20% of the time
in these situations it rained.

Ex: Probability by "relative frequency"

Find Wilt's free throw % if he made
263 of 572 free throws.

$$263/572 = 46\% \approx 45\%$$

Ex: A bag has 72 mm's with 29 red ones.
if an mm is randomly selected find

$$P(\text{red}) = \frac{29}{72} = \approx 403 \quad 40\%$$

3 decimal places

↑ for P = probability

Ex: A sack contains 43 potatoes & 22 onions
If one is randomly selected, find
 $P(\text{onions}) = \frac{\# \text{ onions}}{\text{total}} = \frac{22}{43+22} = .338$

TRY: A bag has 26 red & 41 yellow mm's
Find $P(\text{red}) = \frac{26}{26+41} = .388$

Basic Probability

1. Relative Frequency
2. Space of Equality
3. Complements
4. Certain and Impossible

Experiment: A fair coin is flipped once.
Space of Equality likely outcomes

$\{H, T\}$ (a set)

Find $P(H) = \frac{\# \text{ outcomes that are "H"}}{\text{Total \# of outcomes}} = \frac{1}{2}$



Experiment: A coin is flipped twice.
Find space of equally likely outcomes,
and use it to find $P(\text{at least 1 head})$

$\{ \underline{H} \underline{H}, \underline{H} \underline{T}, \underline{T} \underline{H}, \underline{T} \underline{T} \}$

$$P(\text{at least 1 head}) = \frac{\# \text{ w/ at least 1 Head}}{\text{total}} = P \frac{3}{4}$$

$$P(\text{exactly 2 tails}) = \frac{\# \text{ w/ exactly 2 tails}}{\text{total}} = P = \frac{1}{4}$$

Experiment: 3 Flips

Write S/O and use it to find

$P(\text{exactly 1 Head})$

S/O: $\{ \underline{H} \underline{H} \underline{H}, \underline{H} \underline{H} \underline{T}, \underline{H} \underline{T} \underline{H}, \underline{H} \underline{T} \underline{T}, \underline{T} \underline{T} \underline{T}, \underline{T} \underline{T} \underline{H}, \underline{T} \underline{H} \underline{T}, \underline{T} \underline{H} \underline{H} \}$

$$P(\text{exactly 1 head}) = \frac{\# \text{ w/ exactly 1 Head}}{\text{total}} = P = \frac{3}{8} = .375$$

TRY: Find $P(\text{at least 1 tail}) = 7$

$$\frac{7}{8} = .875$$

$$\text{Find ALL heads} = \frac{1}{8} = .125$$

$P(\text{at least one tail})$ is "complement"

$$\begin{aligned} &\text{The thing to notice } P(\text{All heads}) \\ &+ P(\text{at least one tail}) = \frac{1}{8} + \frac{7}{8} = 1 \end{aligned}$$

Rule of Complements:

A is an event

\bar{A} is a complement

$$\text{rule of complements: } P(A) + P(\bar{A}) = 1$$

Event: $A = \text{It rains}$, $\bar{A} = \text{no rain}$

Example: $A = \text{Wilt makes free throws}$

$\bar{A} = \text{Wilt misses free throws}$

Event: $A = \text{maple Leafs win}$, $\bar{A} = \text{maple loses}$

Example: $P(\text{wilt makes free throws}) = .53$

Find $P(\text{wilt misses})$



$$P(A) + P(\bar{A}) = 1$$

$$.53 + P(\bar{A}) = 1$$

$$P(\text{wilt misses}) = 1 - .53 = .47$$

Example: $P(\text{rain}) = .281$

$$\text{Find } P(\text{doesn't rain}) = 1 - .289 \\ = .719$$

One more thing

Thanksgiving will be on Thursday

= 1 always

$P(\text{certain event, will happen})$

$P(\text{cats fall from sky}) = 0$

$P(\text{impossible event}) = 0$

For all probability: $0 \leq P(A) \leq 1$

6-15-2017
H.W # 6

Probability: Addition Rule

We will use 2-way tables to apply the additional rule:

Peanut Plain

red 14 22

green 61 9

If an mm is randomly selected
Find $P(\text{red or peanut})$

$$= \frac{\# \text{ that are red or peanut (or both)}}{\text{total \#}}$$

$$= \frac{(14 + 22 + 61)}{(14 + 22 + 61 + 9)} = .915$$

Probability:
Addition Rule

HW 6

red yellow

Potatoes 42 116

onions 21 3

$P(\text{yellow or onion})$

$$\frac{(116 + 21 + 3)}{(116 + 21 + 3 + 42)} = .769$$

TRY: Apple Orange

green 51 16

red 23 41

$$\frac{(51 + 23 + 41)}{(51 + 23 + 41 + 16)} = .878$$



"Addition Rule": why is this called?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

↓
or
Union

↓
and
Intersection

$$P(\text{red / apple}) = P(\text{red}) + P(\text{apple}) - P(\text{red and apple})$$

$$P(\text{red}) = \frac{64}{131} + P(\text{apple}) \frac{74}{131} - P(\text{Both}) \frac{23}{131}$$
$$= .878$$

Sometimes, you have to make up the 2-way table from the info. given in the question to apply the additional rule. →

Ex: On a drug test of 117 pos. results, there were 22 false pos. Of 261 neg. results, there were 41 false neg. If a subject is randomly selected, Find: $P(\text{tested neg. or used drugs})$

$$= \frac{41 + 95 + 220}{41 + 95 + 220 + 22} = .942$$

neg. Pos.

used 41 *95

not used *220 (22) ~~41~~

* $117 - 22 = 95$ used

* $261 - 41 = 220$ didn't

TRY: On a drug test on a 125 pos., there were 27 false pos., of 532 neg., there were 22 false neg.

Find: $P(\text{test neg. or didn't use drugs})$

	neg	pos	
used	22	98	$\frac{532+27}{532+125} = \frac{559}{657} = .851$
not used	$\frac{510}{532}$	$\frac{27}{125}$	

Probability: Multiply Rule

1. Dependent & Int+dependent events
2. Acceptance Sampling] applications
3. Redundancy

Coin toss: If you have 2 events, and what happened on the 1st event doesn't affect the probability of the 2nd event, the events are called independent

Ex: 1st coin toss has no effect on the probability of the 2nd coin toss.

If the outcome of the 1st event does affect the probability of the 2nd event, then the events are called dependent.

Ex: Classic Dependent

An urn contains 42 blue marbles and 16 red marbles. If 2 are selected without replacement,

Find: $P(1^{\text{st}} \text{ red}, 2^{\text{nd}} \text{ blue})$

$$\begin{array}{l} 16 \text{ red} \\ + 42 \text{ Blue} \\ \hline 58 \text{ total} \end{array}, \quad \left(\frac{16}{58} \right) \cdot \left(\frac{42}{57} \right) = 0.203$$

\downarrow \downarrow
1st red 2nd Blue

HW #7 Probability: Multiplication Rule

1. Dependent/Independent events
2. Sampling w/ w/o replacement
3. Acceptance sampling
4. Redundancy

A and B are independent if $P(B)$ is not affected by outcome of A and!

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

If $P(B)$ can change depending on outcome of A, then A, B are dependent

$$P(A \text{ and } B) = P(A) \cdot P(B \text{ given } A)$$

$$P(B/A)$$

↓
given

Ex w/o replacement:

A urn contains 44 blue marbles and 61 red marbles. If 2 are randomly selected w/o replacement, find $P(1^{\text{st}} \text{ red}, 2^{\text{nd}} \text{ blue})$

$$\begin{array}{l} 61 \text{ red} \\ + 44 \text{ blue} \\ \hline 105 \text{ total} \end{array} \quad ; \quad \frac{61}{105} \cdot \frac{60}{104} = .335$$

second Red

Ex with Replacement:

100 letters in scrabble of which there are 12 letter "E". If 2 are randomly selected with replacement, Find $P(1^{\text{st}} \text{"E"}, 2^{\text{nd}} \text{"E"})$

$$\frac{12}{100} \cdot \frac{12}{100} \text{ or } (12/100)^2 = .014$$

100 100 pieces

1st E 2nd E

TRY: A bag contains 42 pink starburst and 37 yellow. If 2 are selected w/o replacement Find $P(1^{\text{st}} \text{pink}, 2^{\text{nd}} \text{yellow})$

$$\frac{42}{79} \cdot \frac{37}{78} = .252$$

+37

79

Probability: Acceptance Sampling

EX: Out of 9832 scales in air ports, there are 216 defective. If inspectors test 4 scales w/o replacement

Find $P(4 \text{ good})$

"Combersome calculation"

$$P(1\text{st good}) \cdot P(2\text{nd good}) \cdot P(3\text{rd good}) \cdot P(4\text{th good})$$

9832

- 216

9616 ← good

↓

$$\frac{9616}{9832} \cdot \frac{9615}{9831} \cdot \frac{9614}{9830} \cdot \frac{9613}{9829} = .915$$

← 1 less

5% Rule: Book uses, how to avoid "combersome calculation"

If the sample is less than 5% of the total # of items, you can assume the sampling is done w/ replacement

Ex: Sample was 4 scales, and there were 9,832 total scales.

$$\frac{4 \text{ sample size}}{9832 \text{ total}} = 4.068348251 \times 10^{-4} = .0004$$

way less 5^{10}

if the sample is w/replacement

$$P(4 \text{ good}) = \left(\frac{9616}{9832}\right)^4 \text{ raise 4th power} = .915$$

w/replacement

Fried DO

Ex: A buyer accepts a shipment of bulbs if 5 good ones are randomly selected.

$$P(\text{good one}) = .989 \quad \text{Raise 5th power}$$

$$\text{Find (shipment accepted)} (.989)^5 = .946$$

Ex: A buyer accepts a batch of tires if 4 good ones are randomly selected.

Find $P(\text{batch accepted})$ if $P(\text{bad one}) = .037$

2 steps: Rule of complement

$$\textcircled{1} P(\text{good one}) = 1 - .037 = .963$$

$$\textcircled{2} P(4 \text{ good}) = .963^4 = .86$$

TRY: A shipment of flash drives is accepted if 6 good ones are randomly selected.

Find $P(\text{accepted})$ if $P(1 \text{ bad}) = .008$

Rule of complement

$$\textcircled{1} P(1 \text{ good}) = 1 - .008 = .992$$

$$\textcircled{2} P(6 \text{ good}) = .992^6 = .953$$

↑
6 powers

Good not need Rule of complement

Bad need rule of complement

Redundancy:

Ex: An airplane has an electrical system and a back-up electrical system

$$\text{If } P(\text{electrical works}) = .99$$

$$P(\text{back-up works}) = .987$$

Find $P(\text{both fail})$

$$P(\text{Both fail}) = P(\text{electric fail}) \cdot P(\text{Back-up fail})$$

By rules of complements:

$$P(\text{electrical fails}) = 1 - .99 = .01$$

$$P(\text{Back-up fail}) = 1 - .987 = .013$$

$$P(\text{Both fail}) = (.01)(.013) = 1.3 \times 10^{-4}$$

$$E^{-4} = 10^{-4} = 00013$$

5

$$\text{Ex: } P(\text{1 alarm clock fail}) = .003$$

$$P(\text{2nd alarm clock fail}) = .002$$

You set both alarm clocks

Find $P(\text{Both fail})$

$$= (.003)(.002)$$

$$= 6 \times 10^{-6} = .000006 = 6E-6$$

$$\text{TRY: } P(\text{alarm works}) = .992$$

$$P(\text{alarm 2 works}) = .999$$

Find $P(\text{both fail})$

$$P(\text{alarm 1 fail}) = 1 - .992 = .008$$

$$P(\text{alarm 2 fail}) = 1 - .999 = .001$$

$$P(\text{Both fail}) = (.008)(.001) = 8 \times 10^{-6}$$



Probability: At least one Principle

B-Day problem: 5 people have diff. Birth days

$$\frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365}$$

2pp 3pp 4pp 5pp

At least 1 B-Day match among 5 people is actually the complementary event to no B-Days match

P(no B-Days match in 5 people)

$$1 - \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365} = .0271$$

365⁴ days in year

Ex: P(Wilt makes free throws) = .51
of 5 free throws Find

P(Wilt makes at least 1)

Complement event: Wilt misses All
P(Wilt makes at least 1) = 1 - P(misses All 5)
P(Wilt misses 1) = 1 - .51 = .49 = 1 - .49⁵ (7)
= .972

Probability: A least one principle cont.

$$P(\text{at least 1}) \dots$$

$$= 1 - P(\text{all are the opposite of } \dots)$$

ex: $P(\text{maple Leafs win at least one game of 5}) = 1 - P(\text{they lose ALL 5})$

ex: $P(\text{at least 1 head in 4 flips})$
 $= 1 - P(\text{ALL Tails})$

ex: $P(\text{Wilt passes at least 1 of 7 free throws}) = 1 - P(\text{makes ALL 7})$

ex: If $P(\text{maple Leafs win 1 game of 6 games})$, Find $P(\text{win at least 1})$

① given $P(\text{wins}) = .582$
 Find $P(\text{lose}) = 1 - .582 = .418$

② At least 1 principle:
 $P(\text{at least 1 win}) = 1 - P(\text{lose ALL 6})$
 $= 1 - .418^6 = .995$

Ex: P (Steph makes free throws) = .9
of 5 free throws find the
 P (misses at least 1)

① P (makes) .9
 P (misses) $1 - .9 = .1$

② P (at least 1 miss) = $1 - P$ (make all 5)
 $= 1 - .9^5 = .410$

Ex: P (get right answer) = .25
of 5 questions
find P (at least 1 right)

① P (1 right) = .25
 P (1 wrong) = $1 - .25 = .75$

② P (At least 1 right) = $1 - P$ (5 wrongs)
 $= 1 - .75^5 = .763$

TRY = P (Leafs win 1 game) = .532
of 6 games, find P (Leafs lose at least 1)

① P (win one game) .532
 P (lose one game) $1 - .532 = .468$

② $1 - P$ (win all 6)
 $= 1 - .532^6 = .977$

H.W: #8 Continual Probability

We will use a 2-way table and direct our attention to only the specific parts of the table.

Ex: Plain Peanut

green	16	$\boxed{21}$	
red	9	$+\boxed{60}$	$\frac{21}{21+60} = .259$
		81 total,	

If an mm is randomly selected,
Find $P(\text{green, given it is Peanut})$

only look at Peanut

oval around part of the dark that comes after the word given in the question, only consider what's in the oval to calculate probability

Ex: Thick Thin
light 61 33

Dark 28 74

Elementary Statistics p. 37 Formulas

Birth Weights: Listed below are the weights (grams) of newborn babies from Albany Medical Center Hospital. What value is obtained when those weights are added and total divided by number of weights? (This result called the **mean**, is discussed in ch 3.)

What is noble about these values, and what does it tell us about how the weights are measured?

$$\begin{array}{r} 3600 + 1700 + 4000 + 3900 + 3100 + 3800 + \\ \hline 2200 + 3000 \\ 8 \end{array}$$

Six Children Jule Cole is a founder of Mabel's Labels, and she is the mother of six children. The Probability that six randomly selected children are all girls is found by evaluating 0.5^6

Tallest Person Robert Wadlow (1918-1940) is the tallest known person to have lived. The expression below converts his height of 272 cm to a standard score. Find this value and round the result to two decimal places. Such standard scores are considered to be significantly high if they are greater than 2 or 3. Is the result significantly high?

$$\frac{272 - 176}{6} = \frac{96}{6} = \boxed{16 \text{ yes}}$$

Body Temperature The given expression is used for determining the likelihood that the average (mean) human body temperature is different from the value 98.6°F that is commonly used. Find the given value. Round to two decimal places.

$$\text{std. dev.} \rightarrow \frac{98.2 - 98.6 \leftarrow \text{Population mean}}{\left(\frac{0.62}{\sqrt{106}} \right) \text{ 1st step}} \uparrow \text{sample size}$$

Z-score Formula

$$\text{calculate: } (98.2 - 98.6) / (0.62 / \sqrt{106})$$

$$\text{answer: } -6.64$$

Determining Sample Size

The given expression is used to determine the size of the sample necessary to estimate the proportion of college students who have the profound wisdom to take a statistics course. Find value, round to nearest whole number.

$$\frac{1.96^2 \cdot 0.25}{0.03^2}$$

$$\text{Calculate: } 1.96^2 = 3.8416$$

$$\text{Ans} \times 0.25 = 0.9604$$

$$\text{Ans} / 0.03^2 = 1067.11$$

Answer: 1067

Standard Deviation

The standard deviation is an extremely important concept. Using sample data from Exercise one "Birth weights", part of the calculation for standard deviation is shown in the expression below. Evaluate the expression.

$$\frac{(3600 - 3165.2)^2}{7}$$

Find P (thin, given the sock is dark)

$$= \frac{7}{74+28} = .125$$

9 H.W, Probability Distribution

① Discrete

② Continuous

In any Probability Distribution, there is a random variable X . In a Discrete Distribution, the random variable can take on only fininitely many values, and there is a probability associated with each value. In a Continuous Distribution, the random variable can take on infinitely many values, we calculate the probability the random variable is in an interval.

ex: Discrete Distribution:

X	$P(X)$	$X = \#$ of heads in <u>2</u> flips
0	$1/4$	X could be 0, 1, or 2
1	$2/4$	[HH, HT, TH, TT] coin toss
2	$1/4$	

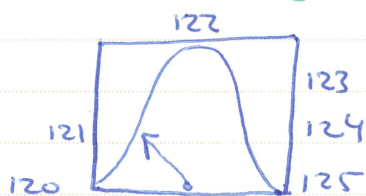
④

note: $\frac{1}{4} + \frac{2}{4} + \frac{1}{4} = 1$

ie $\sum P(x) = 1$

In continuous distribution, the random variable X is measured on some kind of a dial typically.

Ex: $X =$ reading on a volt meter at 2pm



X could be any of the infinitely many #'s in the interval $[120, 125]$

In a continuous distribution we don't calculate the probability that X takes on any particular value, but we calculate probability X is in an interval, such as

$$P(121 < X < 124)$$

One example of Discrete Distribution, the binomial distribution, and our example of a continuous distribution: the normal distribution

In any discrete distribution, the following 2 conditions must be met.

① $0 \leq P(x) \leq 1$ all probabilities must be between 0, 1

② $\sum P(x) = 1$
sum

Ex: Is this a Probability Distribution?

X	P(x)
2	.3
6	.4
9	.3

① $0 \leq P(x) \leq 1$ yes

② $.3 + .4 + .3 = 1$ yes



Ex: Is this a Probability Distribution?

X	P(X)
2	.2
4	.2
6	.2
8	.5

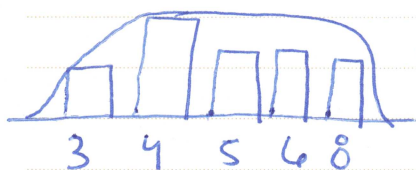
① $0 \leq P(X) \leq 1$ yes

② $.2 + .2 + .2 + .5 = 1.1$ No!

Binomial Distribution

of heads out of 10 flips = 5 (class)

X = # of heads in 10 flips



possible values for X:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Word Questions:

Find P (exactly 4 heads)

P (more than 5 heads)

P (at most 3 heads)

By an unscientific simulation we got,
 P (exactly 4 heads)

$$= \frac{6}{19} = 0.316$$

Formula: P (exactly 4 heads in 10 flips)

$$= \text{binom.pdf}(10, 0.5, 4)$$

calculator:

binom.pdf

trials: 10

p : 0.5

X value: 4

Press Enter = 0.205



6-21-17

Characteristics of a Binomial Probability Distribution

① There are n independent trials
(in our experiment there were 10 flips)

② For each trial, there are 2 possible outcomes, one is called "success", the other is called "failure".

(We call heads a "success" and tails a "failure")

③ for each trial the probability

$$P(\text{Success}) = p$$

$$P(\text{Failure}) = q = 1 - p$$

$$(P(\text{head}) = .5, P(\text{tail}) = .5)$$

④ Random variable $X = \#$ of successes in n trials

($X = \#$ of heads in 10 trials)



This whole set-up is an example of a discrete probability distribution, w/ random variable X taking finitely many values, each w/ an associated probability.

In our experiment!

X	$P(X)$	How can we find $P(X)$?
0		Answer on the calculator
1		we will use: binom.pdf
2		n p X
3		<u># of trials</u> ↓ <u># of success</u>
4		Probability of
⋮		<u>Success</u>
10	$\sum P(X) = 1$	

Ex: A coin is flipped 10 times.
Find $P(\text{exactly 7 heads})$

- ① $N = 10$ # trials
- ② $P = .5$ Prob. of Success
- ③ $X = 7$ # of success

$$= \text{binom.pdf}(10, .5, 7) = .117$$

2nd → DISTR → A: binom.pdf

ex: $P(\text{maple leafs win}) = .426$
of 7 games, Find $P(\text{maple leafs win exactly } 2)$

$n = 7$ trials

$p = .426$ probability success

$X = 2$ # of actual success

$P(\text{maple leafs win } 2 \text{ of } 7)$

$\text{binompdf}(7, .426, 2) = .237$

TRY: $P(\text{pearl in an oyster}) = .163$
of 8 oysters, Find $P(\text{exactly } 3 \text{ pearls})$

$n = 8$ trials

$p = .163$ prob success

$X = 3$ # success

$= \text{binompdf}(8, .163, 3) = .099 = .100$

ex: $P(\text{Wilt makes free throw}) = .51$
of 5 free throws, Find $P(\text{Wilt makes at most } 2) \rightarrow$

X P(X)

0 ✓

n = 5 trials

1 ✓

P = .5 prob. success

2 ✓

X = 2 # success

3	0
4	0
5	+

At most = $P(\text{at most } 2) = P(\text{exactly } 0) + P(\text{exactly } 1) + P(\text{exactly } 2)$

$\text{binom_cdf}(5, .5, 0) + \text{binom_cdf}(5, .5, 1) + \text{binom_cdf}(5, .5, 2) = .481$

We can use a short cut!

$\text{binom_cdf}(n, p, x)$



cumulative

n = # of trials

P = Probability of success

X = Adds probability of success from 0 up to X

$P(\text{at most } 2) = \text{binom_cdf}(5, .5, 2) = .481$



TRY: $P(\text{Leafs win}) = .432$ of
6 games, Find $P(\text{Leafs win at most 3})$

X	P(X)
0	✓
1	✓
2	✓
3	✓

calculate!
2nd → DISTR → Binom pdf
binom pdf (6, .432, 3)
= .774
Answer write on test

4	0
5	T
6	

Binomial Distribution cont.

$$\textcircled{1} P(\text{exactly } X \text{ successes in } n \text{ trials}) \\ = \text{binom pdf } (n, p, X)$$

$$\textcircled{2} P(\text{at most } X \text{ successes in trials}) \\ = \text{binom cdf } (n, p, X)$$

$$\textcircled{3} \text{ Ex: If } P(\text{red pea in a pod}) = .172 \\ \text{Find } P(\text{less than 2 red peas in 7 pods})$$

↓

means $(P \text{ of } '0' + P(1))$

$$= \text{binom cdf } (7, .172, 1)$$

$$\text{Ex: } P(\text{A's win}) = .427$$

On a 6 game road trip, Find
 $P(\text{A's win less than 4 games})$

$$= \text{binom cdf } (6, .427, 3) = .781$$

5 Rules

1. $P(\text{exactly } X \text{ successes}) = \text{binompdf}(n, p, X)$

2. $P(\text{at most } X \text{ successes}) = \text{binomcdf}(n, p, X)$

3. $P(\text{less than } X \text{ successes}) = \text{binomcdf}(n, p, X-1)$

4. $P(\text{more than } X \text{ successes}) = 1 - \text{binomcdf}(n, p, X)$

5. $P(\text{at least } X \text{ successes}) = 1 - \text{binomcdf}(n, p, X-1)$

④ Ex: $P(\text{Wilt makes free throws}) = .51$
of 8 free throws, Find $P(\text{Wilt makes more than } 5)$

6, 7, or 8 free throws

complement to 6, 7, or 8 is

$$0, 1, 2, 3, 4, 5 \Rightarrow \text{binomcdf}(8, .51, 5)$$

By rule of complements

$$P(\text{Wilt makes more than } 5)$$

$$= 1 - P(\text{Wilt makes } 0, 1, 2, 3, 4, 5)$$

$$= 1 - \text{binomcdf}(8, .51, 5) = .158$$

one down

Ex: $P(\text{pearl in oyster}) = .422$
of 11 oysters, Find $P(\text{more than 3 people})$

Complement 1, 2, 3

$$= 1 - \text{binomcdf}(11, .422, 3) = .753$$

calculator

TRY: $P(\text{Shaq makes}) = .527$ of 7
free throws. Find $P(\text{Shaq makes more than 5})$

$$1 - \text{binomcdf}(7, .527, 5)$$

Ex: $P(\text{Leafs win}) = .431$ of 7 games.
Find $P(\text{Leafs win at least 2})$

That means win 2, 3, 4, 5, 6, 7

complement win 0, 1

Rule of Complements:

$$P(\text{Win at least } \boxed{2}) = 1 - P(\text{win } 0, 1)$$
$$= 1 - \text{binomcdf}(7, .431, \boxed{1}) = .878$$

↓

one down

Ex: $P(\text{Stef makes}) = .91$ of 10
free throws. Find $P(\text{Stef makes at least } \boxed{8})$
 $= 1 - \text{binomcdf}(10, .91, \boxed{7}) = .946$



one loss



H.W Help #10 Finish

Ex: An airplane that 94% of people who buy a ticket actually show up. If an airline sells 25 tickets with 23 seats, find the probability (not enough seats)

$$= P(\text{more than 23 successes})$$

$$= 1 - \text{binomcdf}(25, .94, 23) = .553$$

Ex: (acceptance sampling) In a large batch of aspirin $P(\text{good one}) = .999$.

A batch is accepted if among 6 aspirin, there is at most 1 bad. Find $P(\text{batch accepted})$

Bad one is success, so $P(\text{success}) = 1 - .999 = .006$
 Rule of complements:

$$P(\text{at most 1 success}) = \text{binomcdf}(6, .006, 1) = .999$$

TRY: A batch is accepted if among 15 tires there are at most 2 bad.

Find $P(\text{batch accepted})$

$$n=15, p=.007, X=2; (15, .007, 2) = .999$$

: binomcdf

Binomial Distribution H.W 11

A random variable X that has the binomial distribution, has a mean, (expected value) μ = mean and a standard deviation σ = sigma

$$\mu = np = (\text{\# of trials}) \cdot (\text{probability of success})$$

$$\sigma = \sqrt{npq}, \quad q = 1 - p$$

Range of usual value (rouv)
 $= [\mu - 2\sigma, \mu + 2\sigma]$

ex: X = # of heads in 10 flips, Find
 μ, σ, rouv

$$\mu = np = 10(0.5) = 5$$

$$\sigma = \sqrt{npq} = \sqrt{10(0.5)(0.5)} = 1.5 \rightarrow 1.6$$

$$\text{rouv} = [\mu - 2\sigma, \mu + 2\sigma]$$

$$= [5 - 2(1.6), 5 + 2(1.6)]$$

$$= [1.8, 8.2]$$

Ex: $P(\text{Giants win}) = .423$ of 162 games.
Find μ , σ , rouv

$$\mu = np = 162(.423) = 68.5$$

$$\sigma = \sqrt{npq} = \sqrt{162(.423)(.577)} = 6.3$$

$$\text{rouv} = [68.5 - 2(6.3), 68.5 + 2(6.3)]$$

55.9       81.1

TRY: $P(\text{Pearl in oyster}) = .241$ of 18 oysters. Find μ , σ , rouv

$$\mu = np = 18(.241) = 4.3$$

$$\sigma = \sqrt{npq} = \sqrt{18(.241)(.759)} = 1.8$$

$$\text{rouv} = 4.3 - 2(1.8), 4.3 + 2(1.8)$$

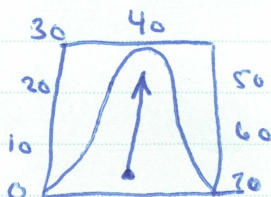
.7       7.9

$$\mu = np = 18(.241) = 4.3$$

$$\sigma = \sqrt{npq} = \sqrt{np(1-p)} = \sqrt{18(.241)(.759)} = \sqrt{1.8}$$

Continuous Probability Distribution

Ex: The reading on the speedometer of my old '66 Dart



X = any # between '0' and 70
could be 55, $\sqrt{7}$ any real # between
0 and 70. Infinitely many possibilities.

recall: Discrete X we had a table:

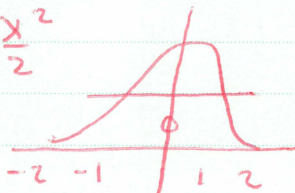
X	$P(X)$
1	.4
2	.2
3	.4
	<hr/>
	1

$\sum P(X) = 1$

For a continuous random variable we don't speak about the probability it takes on any particular value, we speak of the probability that it is inside some interval.

In stead of the table of X and $P(X)$ we have a "probability distribution function" (pdf) and probabilities are calculated as geometric areas bounded by the probability distribution function.

Although there are many different probability distribution function, we will focus on the standard normal distribution function.

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$


Graph comment: Graph has the X -axis as a horizontal asymptote in both directions.

Graph is symmetric w/ respect to y -axis

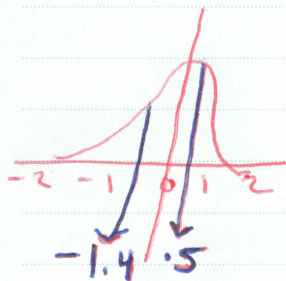
and: If X is standard normal random variable, we calculate probabilities associated w/ X as geometric area under the standard normal distribution function.

Ex: If X is a standard normal,
Find $(0 < X < 1)$

Normal cdf $(0, 1, 0, 1) = .341$
upper x boundary
lower x boundary
mean
std. dev.

Ex: X is a Standard normal

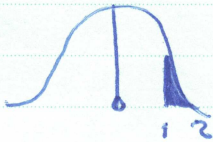
Find $P(-1.4 < X < .5) = \text{normal cdf}(-1.4, .5, 0, 1)$
 $= .611$



Standard Normal Distribution

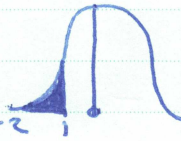
1. Normalcdf (low X, high X, μ , σ)
2. Percentiles
3. Critical z-value

Ex: X is std. normal. calculator
 Find $P(1 < X < 2) = \text{normalcdf}$
 $(1, 2, 0, 1) = .136$



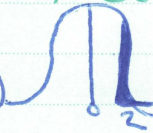
Ex: X is a standard normal
 $P(-2 < X < -1.5)$

normalcdf(-2, -1.5, 0, 1) = .044



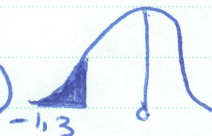
Ex: X is a standard normal
 $P(X > 2)$

normalcdf(2, 999, 0, 1) = .023

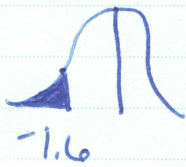


Ex: X is standard normal
 $P(X < -1.3)$

normalcdf(-999, -1.3, 0, 1) = .097



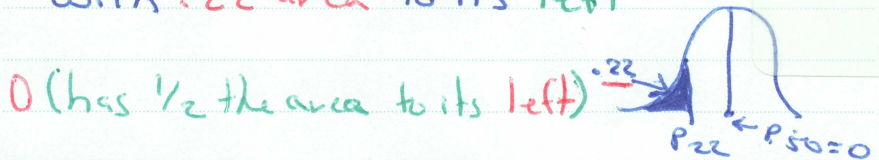
TRY: X is Std. Normal
 Find $P(X > -1.6)$



normal cdf $(-1.6, -999, 0, 1)$
 $= .945$

So far, I've given you the X -
 boundary values, you find the probability
 as an area.

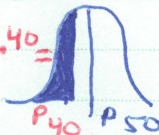
Probabilities $P_{.22} =$ value along X -axis
 with $.22$ area to its left



Calculator $P_{.22} = \text{inv Norm}(.22, 0, 1) = -.77$

\leftarrow area \leftarrow \downarrow μ \rightarrow σ
 $P_{.22}$ \leftarrow $P_{.50} = 0$

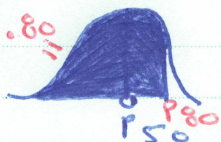
Ex: X stand. normal

Find $P_{.40}$ $.40 =$  $\text{inv Norm}(.4, 0, 1)$
 $= -.25$



Ex: X is Stand. Normal

Find $P_{80} = .80$

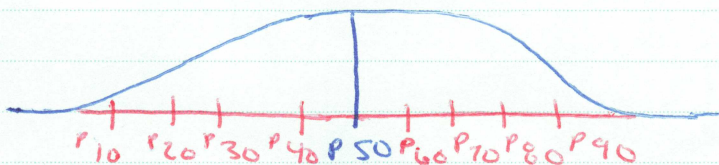


$$\text{InvNorm}(.8, 0, 1) = .84$$

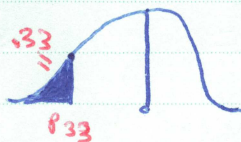
If the percentile is less than 50 \Rightarrow
 X -value on negative side, the smaller
the percentile the more negative it is.

If the percentile is greater than 50 \Rightarrow
 X -value is on positive side, larger the
percentile the more positive it is.

If percentile = 50 $\Rightarrow X = 0$ (for stand.
normal)

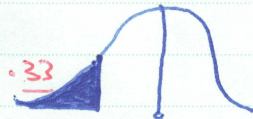


Ex: Find $P_{33} = .33$



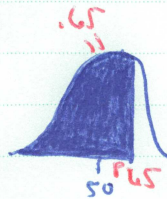
$$\text{InvNorm}(.33, 0, 1) = .44$$

Turn it around: If X is Stad. Normal
Find $P(X < -0.44)$



$$-0.44 = \text{normal_cdf}(-999, \dots, -0.44, 0, 1)$$
$$= \underline{.33}$$

TRY: Find $P_{.65}$



$$\text{InvNorm}(.65, 0, 1)$$
$$= .39$$

Area to Right, Critical z-value

$Z_\alpha = Z$ -value w/ area α to its **Right**
alpha \rightarrow

Why are we calling this a Z -value when it's relating to the Stand. Normal distribution w/ $\mu = 0$ $\sigma = 1$

recall! $Z = \frac{X - \bar{X}}{S}$ sample data

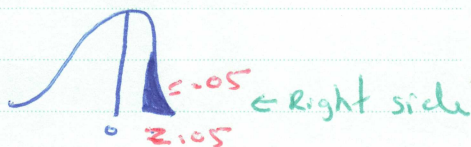
$Z = \frac{X - \mu}{\sigma}$ population data

$Z = \frac{X - 0}{1} = X$ for Standard Normal

So if X is a Standard Normal, $X = Z$, for critical Z -values, we're still talking about the same old Standard Normal X .

$Z_\alpha = Z$ -value w/ area α to its **Right** and typically α is a small value like $.05$ or $.01 \dots$

Ex: Find $Z_{.05}$



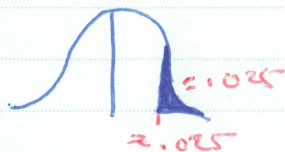
If $.05$ is area to Right than $1-.05$ is area to the Left, and that's what we need for Inv Norm

$$Z_{.05} = \text{InvNorm}(1-.05, 0, 1) = 1.64$$

↑
no comma

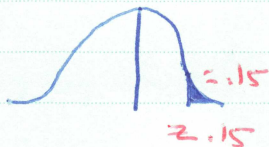
i.e. FLIP normalcdf(1.64, 999..., 0, 1)
= .05

Ex: Find $Z_{.025}$



$$\text{Inv Norm}(1-.025, 0, 1) = 1.96$$

TRY: Find $Z_{.15}$

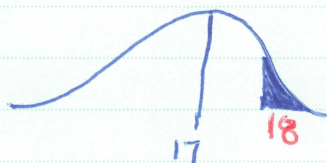


$$\text{Inv Norm}(1-.15, 0, 1) = 1.04$$

↑
no comma

Next item: X will be a normally distributed random variable (not standard normal)

If height of a giraffes X is normally distributed with $\mu = 17$ ft., std dev. $= 2.4$ ft. Find probability a randomly selected giraffe is taller than 18 ft.



normalcdf(18, 999..., 17, 2.4)
 $= 33\%$

Normal Distributions

1. Standard Normal ($\mu=0, \sigma=1$)

a. normalcdf

b. percentil

c. Critical z-value } invnorm

2. Any normal distribution

a. normalcdf

b. Invnorm

3. Central Limit Theorem

Ex: Weight of elephants X is normal distributed mean = 2100 lbs., std. dev. = 125 lbs. If an elephant is randomly selected,

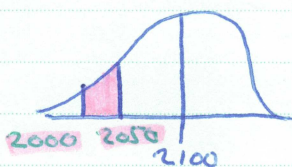
Find $P(2000 < X < 2050)$

Formula: std. normal

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

in general: $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

normal
cdf



$$P(2000 < X < 2050)$$

$$= \text{normalcdf}(2000, 2050, 2100, 125)$$

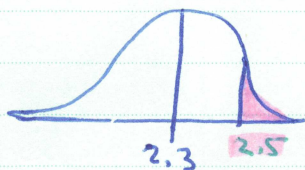
low high μ σ

$$= .133$$

Ex: The height X of raccoons is normally distribution

$$\mu = 2.3 \text{ ft.}$$

$$\sigma = .27 \text{ ft.}$$



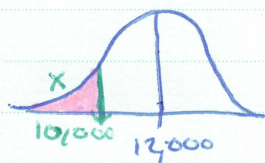
Find $P(X > 2.5)$

$$= \text{normalcdf}(2.5, 999.111, 2.3, .27) = .229$$

low high μ σ

Ex: The weight of planets X is normally distributed with mean = 12,000 whatever std dev. 1,500

Find: $P(X < 10,000)$ less = neg.



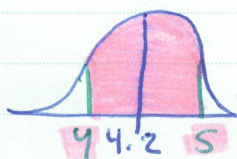
$$\text{normalcdf}(-999.999, 10,000, 12,000, 1,500) \\ \approx .091 \text{ less}$$

TRY: The height of porcupines X is normally distributed

$$\mu = 4.2 \text{ ft}$$

$$\sigma = .8 \text{ ft} \quad \text{Find } P(4 < X < 5)$$

$$\text{normalcdf}(4, 5, 4.2, .8) = .440$$

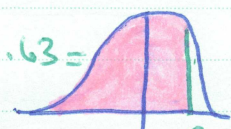


Ex! A standardized test score is normally distributed with

$$\mu = 100$$

$$\sigma = 15$$

Congrats! Your child was in the 63rd percentile (P63) What was the score?



$$P_{.63} = \text{invnorm}(.63, 100, 15) = 105.0$$

area to left
mean
std. dev.

P = Left, Z = Right

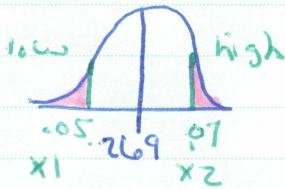
Ex: Length of pregnancy is normally distributed

$$\mu = 269 \text{ days}$$

$$\sigma = 6 \text{ days}$$

A hospital has to give special care for babies in the lowest 5% and highest 7% of this distribution.

Find the cutoff point. →



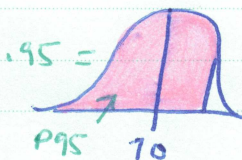
$$x_1 = \text{invnorm}(.05, 269, 6) = 259.1$$

$$x_2 = \text{invnorm}(1 - .01, 269, 6)$$

$\downarrow = 277.9$
 Flip side area to
 (L)

Ex: men's heights are normally distributed with $\mu = 70$ in.
 $\sigma = 5$ in

An airline, it's entrance height to admit 95% of men w/o hitting their heads. How high should it be?



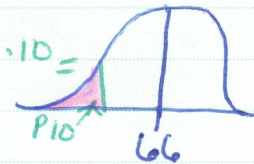
$$= \text{invnorm}(.95, 70, 5) = 78.2$$



TRY! Woman's heights are normally distributed $\mu = 66$ in
 $\sigma = 3.5$ in

The army will not accept recruits in the lowest 10% of all women's heights.

What's the minimum height for a female recruit?



$$\text{invnorm}(.10, 66, 3.5) = 61.5$$

The Central Limit Theorem

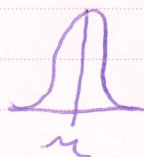
If a population is normally distributed w/ mean = μ , std dev. = σ and sample of size n are randomly selected, and the mean of each sample is computed, then:

- ① the mean of the sample means denoted $\mu_{\bar{x}} = \mu$ (equals the population mean)
- ② The standard deviation of the sample means, denoted $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ (population dev. divided)

If $n > 30$, the original population need not be normally distribution theorem to be true.



original population
(not nvc)

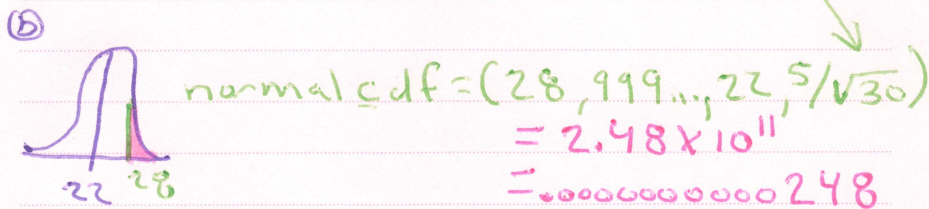
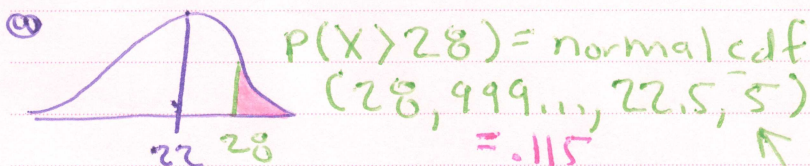


Less dispersed, this population
(of classes) (smaller std. dev.)

Ex: UVC students ages are normally distributed w/ mean = 22, $\sigma = 5$

(a) Find $P(X > 28)$

(b) If sample of size 30 are selected, Find probability mean age of a sample is > 28

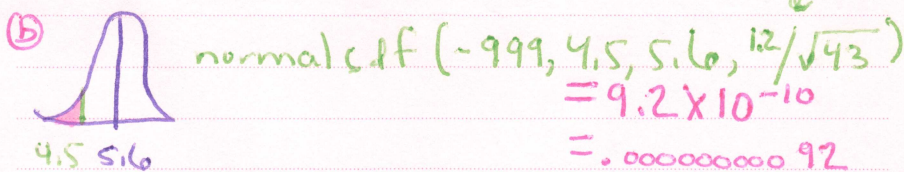
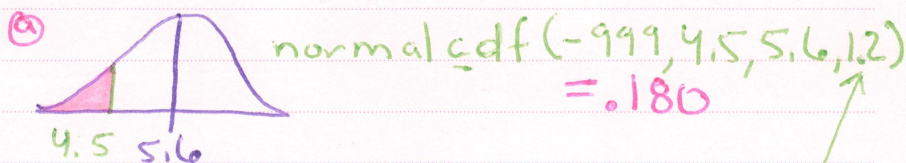


i.e. The change chance of an "outlier" in ways less for the sample means than it is for the individuals in the original population.

Ex: Height of aliens is normally distributed w/ $\mu = 5.6$ ft
 $\sigma = 1.2$ ft

(a) If a single alien is selected
Find $P(\text{less than } 4.5 \text{ ft})$

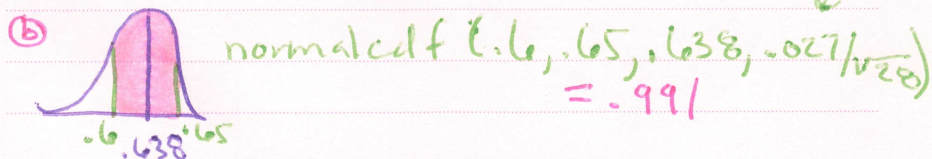
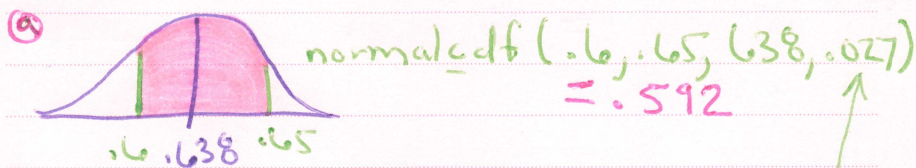
(b) If a sample of 43 aliens is selected,
Find $P(\text{mean height} < 4.5 \text{ ft})$



Ex! (Sample means more centrally located) weight of gnats is normally distributed $\mu = .638$
 $\sigma = .027$

(a) If a single gnat is selected,
Find $P(\text{its weight is between } .63 \text{ and } .65)$

(b) If sample of 28 gnats is selected
Find $P(\text{mean is between } .6 \text{ and } .65)$



7.4.2017

Inferential Statistics

HW 15

- ① Confidence Intervals (CI)
- ② Hypothesis Testing

CI

- ① Population proportions
- ② Population mean, σ known
- ③ Population mean, σ unknown

In a sample of 29 people 11 believe in ghosts construct a 95% CI containing the population proportion of people who believe in ghosts.

\hat{P} = sample proportion

P = population proportion

$$\hat{p} = \frac{11}{29} = 0.379$$

Sample proportion \hat{P} is the best "point estimate" for P , the population proportion.

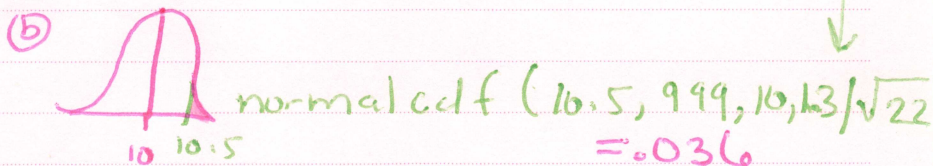
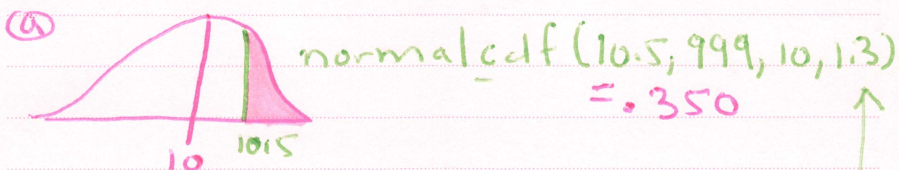
We will construct a CI, using \hat{p} as its center, of the form $(\hat{p} - E, \hat{p} + E)$ where E = margin of error that actual P is inside the CI.

①

TRY: Weight of cats is normally distributed $\mu = 10$ lbs
 $\sigma = 1.3$ lbs

(a) If a cat is selected, Find
 $P(\text{weights more than } 10.5 \text{ lbs})$

(b) If a sample of 22 cats is selected,
 $P(\text{mean weight more than } 10.5)$

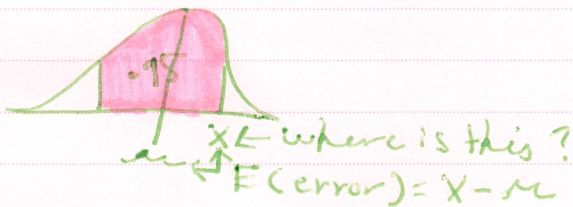


Theory behind CI

Samples of size n are normally distributed w/ mean = population mean

σ = standard deviation

These will vary depending on what kind of CI.



$$z = \frac{x - \bar{x}}{s} \quad (\text{sample data})$$

$$= \frac{x - \mu}{\sigma} \quad (\text{population})$$

$$z = \frac{E}{\sigma} \quad (E = x - \mu)$$

$$E = z\sigma$$

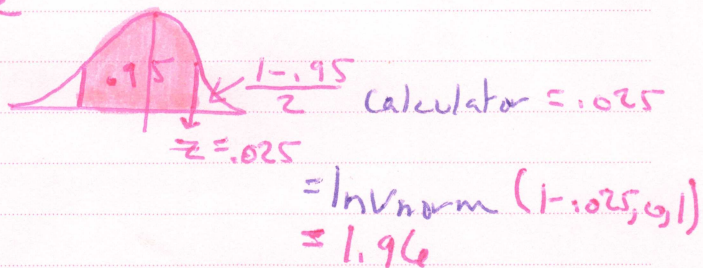
The error is equal to the appropriate Z -value times the appropriate standard deviation. The appropriate Z -value depends on what % CI you are constructing; and appropriate std. dev. depends on what kind of CI you are constructing (population proportion or population mean)

For population proportion, the appropriate formula for std. dev.

$$\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

\hat{p} = sample proportion
 $\hat{q} = 1 - \hat{p}$
 n = sample size

For a 95% CI, the appropriate Z -value



Ex: Construct 95% CI containing population proportion if 11 of 29 people believe in ghosts.

$$\textcircled{1} \hat{p} = \frac{11}{29} \text{ calculate} = .379$$

$$\textcircled{2} \hat{q} = 1 - \hat{p} = 1 - .379 = .621$$

$$\textcircled{3} n = 29$$

$\textcircled{4}$ z:



$$.025 = \text{invnorm}(-1, .025, 0) \\ = 1.96$$

$$\textcircled{5} E = z \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96 \sqrt{\frac{.379(.621)}{29}} = \boxed{.177}$$

$$* 1.96 * \sqrt{(.379 * .621) / 29} = \boxed{.177} \leftarrow \text{calculator}$$

$$\textcircled{6} (CI = \hat{p} - E, \hat{p} + E)$$

$$= (.379 - .177, .379 + .177) \text{ three decimal} \\ = \boxed{.202}, \boxed{.556}$$

✓ on calculator
Stat → test

↓

A 1-Prop Z-Int

X: 11

N: 29

C-level: .95

Calculate

(.20271, .55591)

$\hat{p} = .3193...$

n = 29

✓

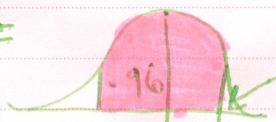
Ex! Construct a 96% CI containing population proportion of 262 of 1123 people have seen a UFO.

① $\hat{p} = \frac{262}{1123} = .233$ ← sample portion

② $\hat{q} = 1 - \hat{p} = 1 - .233 = .767$

③ n = 1123

④ z =



$\frac{1 - .96}{2} = .02$

$z = .02 = \text{invnorm}(1 - .02, 0, 1)$

$= 2.05$

⑤

$$\textcircled{5} E = Z \sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.05 \sqrt{\frac{.233(.767)}{1123}} = \boxed{.026}$$

$$2.05 * \sqrt{.233 * .767 / 1123} = .026 \quad \boxed{\text{calculator}}$$

$$\textcircled{6} CI = (\hat{p} - E, \hat{p} + E)$$

$$= (.233 - .026, .233 + .026)$$

$\boxed{.207} \qquad \boxed{.259}$

✓ STAT → TEST
↓

1 prop Z Int:

X: 226

n: 1123

C-level: .96

calculate

(.20738, .25922)

$\hat{p} = .233$

$n = 1123$

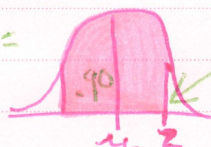


Ex: Construct 90% CI for population proportion if 862 of 1477 people support prop # 216, 172

$$\textcircled{1} \hat{p} = \frac{862}{1477} = \boxed{.584}$$

$$\textcircled{2} \hat{q} = 1 - .584 = \boxed{.416}$$

$$\textcircled{3} n = 1477$$

$$\textcircled{4} z = \frac{1 - .9}{2} = .5$$


$.05 = \text{invnorm}(1 - .05, 0, 1) = 1.64$

$$\textcircled{5} E = z \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 1.64 \sqrt{\frac{.584(.416)}{1477}} = .021$$

$$\textcircled{6} \text{CI} (\hat{p} - E, \hat{p} + E)$$

$$= (.584 - .021, .584 + .021)$$

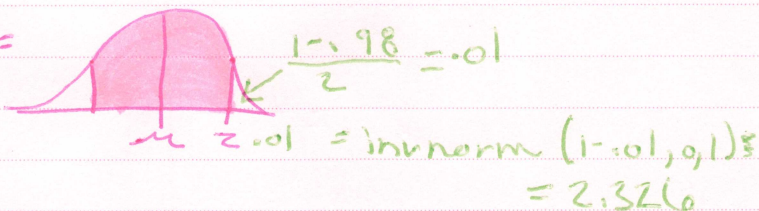
$\boxed{.563}$ $\boxed{.605}$

TRY: Construct 98% CI if 462 of 1135 have seen a USO.

$$\textcircled{1} \hat{p} = \frac{462}{1135} = .407$$

$$\textcircled{2} \hat{q} = 1 - .407 = .539$$

$$\textcircled{3} n = 1135$$

$$\textcircled{4} z =$$


$\frac{1 - .98}{2} = .01$
 $z = 0.01 = \text{invnorm}(1 - .01, 0, 1) = 2.326$

$$\textcircled{5} E = z \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 2.33 \sqrt{\frac{.407(.539)}{1135}} = 0.034$$

$$\textcircled{6} \left. \begin{array}{l} .407 - .034 = .373 \\ .407 + .034 = .441 \end{array} \right\} (.373, .441)$$

Ex: How many people should be surveyed to construct a 95% CI with error = 3% if p is estimated a .423

$$E = Z \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$\frac{E}{Z} = \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$\frac{E^2}{Z^2} = \frac{\hat{p}\hat{q}}{n}$$

$$nE^2 = Z^2 \hat{p}\hat{q}$$

$$n = \frac{Z^2 \hat{p}\hat{q}}{E^2}$$

Formula: $n = \frac{Z^2 \hat{p}\hat{q}}{E^2}$

$Z =$



$$\frac{1 - .95}{2} = .025$$

$$\approx z \cdot 0.025 = \text{invnorm}(1 - .025, 0, 1)$$
$$= 1.96$$

$\hat{p} = .423, \hat{q} = 1 - \hat{p} = .577, E = .03$

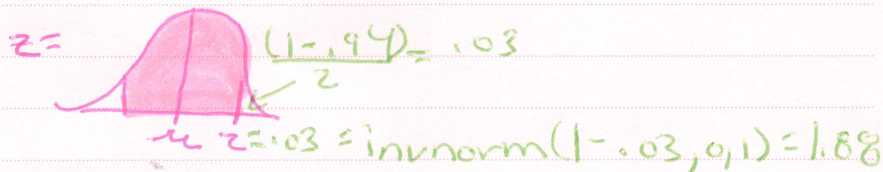
$$\checkmark$$

$$\text{Formula: } n = \frac{(1.96)(.423)(.577)}{(.03)^2} = 10.42$$

Round up to 11

If no estimate for \hat{p} is given use
 $\hat{p} = \hat{q} = .5$

ex: How many should be surveyed to
 construct a 94% CI with error = 2%



$$n = \frac{z^2 \hat{p} \hat{q}}{E^2} = \frac{1.88^2 (.5)(.5)}{.02^2} = \boxed{2209}$$

CI cont.

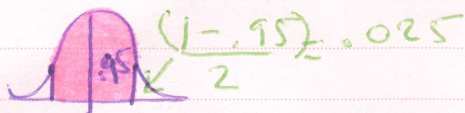
- ① proportions
- * ② population means (σ known)
- ③ population means (σ unknown)

Ex! Construct 95% CI containing the population mean height of giraffes is a sample of 41 giraffes has mean height = 17.2 ft. and the population std. dev. for all giraffes is 1.7 ft.

$$CI: (\bar{X} - E, \bar{X} + E)$$

$$E = z \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{X} = 17.2 \quad n = 41 \quad \sigma = 1.7 \quad z = \downarrow$$



$$z_{.025} = \text{invnorm}(1 - .025, 0, 1) = 1.96$$

$$E = z \cdot \frac{\sigma}{\sqrt{n}} = 1.96 * 1.7 / \sqrt{41} \leftarrow \text{calculator}$$

$$= 1.96 \cdot \frac{1.7}{\sqrt{41}} = .52$$

→

$$\text{CI: } \bar{X} - E, \bar{X} + E = 17.2 - .52 = 16.68$$
$$17.2 + .52 = 17.72$$

Calculator: STAT \rightarrow TEST

\downarrow

ZInterval \rightarrow stats

$$\sigma: 1.7$$

$$\bar{X}: 17.2$$

$$C\text{-level} = .95$$

calculate

$$(16.68, 17.72)$$

$$\bar{X} = 17.2$$

$$n = 41$$

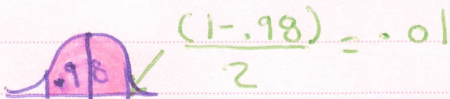


Ex! Constructs 98% CI containing the mean number of choco. chips in a cookie if a sample of 33 cookies has $\bar{X} = 20.6$ and population std. dev. for all cookies is 5.3.

$$CI = \bar{X} - E, \bar{X} + E$$

$$E = Z \frac{\sigma}{\sqrt{n}}$$

$$\bar{X} = 20.6 = 5.3 \quad n = 33 \quad Z = 2$$



$$Z_{.01} = \text{invnorm}(1 - .01, 0, 1) = 2.33$$

$$E = Z \cdot \frac{\sigma}{\sqrt{n}} = 2.33 * 5.3 / \sqrt{33}$$

$$2.33 \cdot \frac{5.3}{\sqrt{33}} = 2.15$$

$$CI = \bar{X} - E, \bar{X} + E$$

$$20.6 - 2.15 = 18.45$$

$$20.6 + 2.15 = 22.75$$

Stat \rightarrow test \downarrow

Z Invr \rightarrow STATS ... Calculate

$\sigma = 5.3$ $\bar{X} = 20.6$ $n = 33$ C-level ...

(18.454, 22.746) $\bar{X} = 20.6$ $n = 33$ ③

TRY! Construct 90% CI containing population mean weight of gnats if a sample of 52 gnats has $\bar{x} = .265$ and population std. dev. for weight of all gnats is .053

$$\bar{x} = .265 \quad \sigma = .053 \quad n = 52 \quad z_{\alpha/2}$$



$$z_{\alpha/2} = z_{.05} = \text{invnorm}(1 - .05, 0, 1) = 1.64$$

$$E = 1.64 \frac{(.053)}{\sqrt{52}} = .012$$

$$.265 - .012 = .253$$

$$.265 + .012 = .277$$

* recopy notes

Formula

wrong answer

$$E = \frac{z\sigma}{\sqrt{n}}$$

$$E\sqrt{n} = z\sigma$$

$$\sqrt{n} = \frac{z\sigma}{E}$$

$$n = \frac{z^2 \sigma^2}{E^2} = \left(\frac{z\sigma}{E} \right)^2$$

Ex: How many elephants should be weighed to construct a 95% CI with $E = 40$ lbs and the population std. dev. for all elephants is 150 lbs.

$$n = \frac{z^2 \sigma^2}{E^2}$$



$$z_{.025} = \text{invnorm}(1 - .025, 0, 1) = 1.88$$

1.96

$$n = \frac{1.88^2 (.25)^2}{(.05)^2} \left[\frac{1.88^2 * .25^2}{.05^2} \right] = 89$$

roundup - no 1/2

σ Unknown

Ex! Construct 95% CI containing mean height of aliens if a sample of 41 aliens has mean height = 5.2 ft
std. dev. = 0.43 ft

Recall: Before $E = Z \frac{\sigma}{\sqrt{n}}$

"Student distribution"

2nd \rightarrow Distr
 \downarrow
pdf

CI, means $\rightarrow \sigma$ unknown

When σ is not given, the sample std. dev. is given, and instead of using a Z-value, we will use a T-value and the formula for error

$$E = \frac{tS}{\sqrt{n}} \quad \left(\text{Compare to } E = Z \cdot \frac{\sigma}{\sqrt{n}} \right)$$

Student t-distribution

A family of distribution on "degree of freedom" $df = n - 1$ (always one less than sample size)

On calculator:

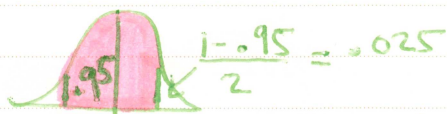
Invt (area to the left, df, degree freedom)

Ex: Construct 95% CI containing population mean weight of raccoons if a sample of 26 raccoons has a mean weight = 14.6 lbs. std dev = 1.3 lbs

$$CI (\bar{X} - E, \bar{X} + E)$$

$$E = \frac{tS}{\sqrt{n}} \quad \bar{X} 14.6 \quad S 1.3 \quad n 26$$



$t =$ 

$$t_{.025} \text{ InvT}(1 - .025, 25) = 2.060$$

$\text{Dist} \rightarrow \text{invT} \uparrow$
 \downarrow

$$df = n - 1, n = 26$$

$$\frac{tS}{\sqrt{n}} = \frac{(2.06)(1.3)}{\sqrt{26}} = .53$$

$$* 2.06 * 1.3 / \sqrt{(26)}$$

$$CI = \bar{X} - E, \bar{X} + E$$

$$(14.6 - .53, 14.6 + .53)$$

14.07
15.13

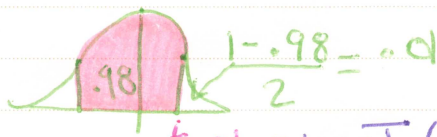
Ex! Construct 98% CI containing mean weight of cream-filling in Twinkies is a sample of 29 Twinkies has cream filling with mean weight = .83
 std dev = .06

$$CI (\bar{X} - E, \bar{X} + E)$$

$$\begin{array}{r}
 X .83 \\
 E = \frac{tS}{\sqrt{n}} \quad S .06 \\
 \quad \quad \quad n \quad 29
 \end{array}$$

(2)

t =



$$t_{.01} = \text{invT}(1 - .01, 28) = 2.467$$

$$E = \frac{tS}{\sqrt{n}} = \frac{(2.467)(.06)}{\sqrt{29}} = .027$$

$$\bar{X} - E, \bar{X} + E$$

$$(.83 - .027, .83 + .027)$$

.803.857

✓ STAT → TEST → Tinterval → Stats

$$(.803, .857)$$

$$\bar{x} = .83$$

$$s_x = .06$$

$$n = 29$$

Hypothesis Testing

① Proportion

② mean

- σ Known
- σ Unknown

③ 2 proportions

④ 2 independent means

⑤ 2 dependent means (matching pairs)

⑥ Correlation regression

Steps of Hypothesis Testing

- Set it up
- Calculator for proper test
- Interpret results

11 of 36 have seen a U.F.O

ex: With $\alpha = .05$, test the claim more than .25% of all people have seen a U.F.O if 11 of 36 have seen a U.F.O.

① Set-up

Claim: $P > .25$ (P = population proportion)
opposite of claim: $P \leq .25$

null hypothesis

↓

$H_0: P = .25$ (not from original claim because original claim has no $=$ sign)

alternate hypothesis

↓

$H_1: P > .25$ (one of the claim or its
Right-tail test opposite that does not contain $=$ sign)

② Calculator - 1 prop z test

STATS → TESTS

↓

1 prop z test

$P_0: .25$ (p-value from H_0)

$X: 11$

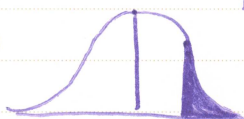
$n: 36$

prop: $\neq p_0$ $< p_0$ $> p_0$ enter
match form of H_1

Draw

Interpretation

③



$$Z = .7698$$



$P = .2207$ → All important "p-value"

test statistic, used in "critical reason"

The "P-value" is probability of obtaining sample data at least as ~~extreme~~ extreme as the given sample data if H_0 is actually true.

If the "P-value" is really low, then the probability of obtaining the sample data assuming H_0 is true is very low, makes ^{us} think about saying H_0 is not true, "reject H_0 "

$\alpha = .05$ (α is called the "significance of the test") is the cut-off for rejecting H_0 .

If $P < \alpha \Rightarrow$ reject H_0 if P is not $< \alpha =$ fail to reject H_0 in our example $p = .2207$ since $.2207$ not less than $.05 \Rightarrow$

③

① Fail to reject H_0

② H_0 not from original claim
(from set-up) from chart

"not sufficient evidence to support claim"⁴

ex! With $\alpha = .05$, test claim
at most 40% of people support
X, if 395 of 947 support X.

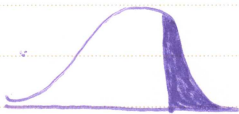
Claim $p \leq .4$

opp $p > .4$

H_0 $p = .4$ (from original claim)

H_1 $p > .4$ (right-tail)

Calculate \rightarrow prop z test



$z = 1.0746$ $P = .1413$

Hand out: Not sufficient evidence
to support claim ...
reject

Since " P " is not less than $\alpha \Rightarrow$ fail to
reject H_0 , H_0 from original claim

④

Hypothesis Testing Algorithm

① Set-up

(A) claim (B) opposite (C) H_0 (D) H_1

② Calculator

STAT \rightarrow TEST (appropriate test)

\downarrow

Enter data \rightarrow Draw picture

③ Interpretation

④ If $p < \alpha \Rightarrow$ reject H_0 if not, fail to reject H_0

⑤ Write conclusion from chart

Ex \rightarrow

H.W # 17, Book 409/17

Ex: With $\alpha = .05$, test claim less than 33% of tennis challenges are successful if 172 of 611 are successful.

claim $p < .33$

opp $p \geq .33$

H_0 $p = .33$ (not original)

H_1 $p < .33$ (left-tail test)

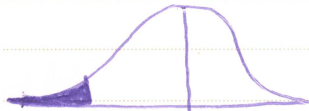
Calculate 1 prop z test

P_0 .33

X 172

n 611

prop $< p_0$ enter \rightarrow Draw



$z = -2.5493$ $P = .0054$

Since $p < \alpha \Rightarrow$ reject H_0 not original
 \Rightarrow data support claim ...

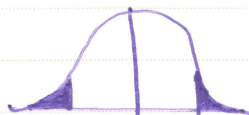
Ex! With $\alpha = 0.05$ test claim proportion who thought they voted for winning candidate is .43 if 308 out of 611 thought so.

Claim $p = .43$ opp $p \neq .43$ $H_0 p = .43$
 $H_1 p \neq .43$ 2-tail From org.

1 prop test

$p = .43$ $X = 308$ $n = 611$ prop $\neq p_0$

Enter



$$\begin{aligned} z &= 3.6993 & p &= 2E-4 \\ & & &= 2 \times 10^{-4} \\ & & &= .0002 \end{aligned}$$

$P < \alpha \Rightarrow$ reject H_0 original \Rightarrow
Sufficient data to reject claim...

TRY: Proportion who have been abducted by aliens is less than 5% if 27 out of 962 claim to have been abducted.

with $\alpha = .05$ test claim

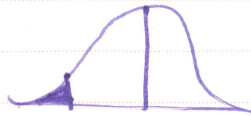
claim $.05 > p$

opp $p \geq .05$

H_0 $p = .05$ not org

H_1 $p < .05$ left

1 prop z test



$$p = 9E^{-4}$$

$$p = 9 \times 10^{-4}$$

$p < \alpha \Rightarrow$ reject H_0 , not org.

\Rightarrow Data support claim

Hypothesis Test cont.

H.W 18

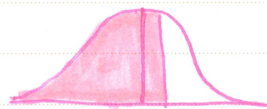
mean

 σ known (z-thing) σ unknown (t-thing)Use proper notation on testIf the population standard dev is given \Rightarrow use Z-testIf sample dev is given \Rightarrow use T-test
Use μ instead of P when setting up the testex: with $\alpha = .05$ test the claim mean ft. of giraffes is at least 17ft. if a sample of 41 giraffes has mean ft = 17.55 ft and std dev = 1.3 ft. $\mu \rightarrow$ population mean $P \rightarrow$ population proportion

population mean

T-Test STAT \rightarrow TEST μ_0 17.5 \bar{x} 17.55 s_x 1.3 n 41 $\mu < \mu_0$ \downarrow

TTest

 \downarrow Stats $t = .2463$ $P = .5966$

①

P not less than $\alpha \Rightarrow$

fail to reject H_0 from org. claim
not sufficient evidence to reject claim.

Book: 421/13 Ex!

With $\alpha = .05$ test the claim the mean weight of mm's is .8535g.
if a sample of 19 mm's mean weight = .8635 and std. dev for weight for ALL mm's is .057g.

Claim $\mu = .8535$

opp $\mu \neq .8535$

H_0 $\mu = .8535$ (org. claim)

H_1 $\mu \neq .8535$

Calculator z-test: STAT \Rightarrow TEST

$\mu_0 = .8535$

$\sigma = .057$

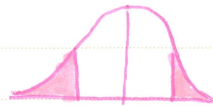
$\bar{x} = .8635$

$n = 19$

$\mu \neq \mu_0$

Z-TEST

Draw



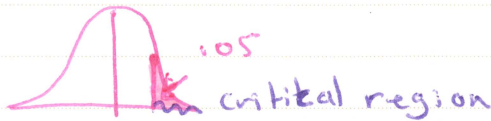
$z = .7647$ $[P = .4444]$

(2)

P is not less than α , fail to reject H_0
from org. \Rightarrow not suff. evidence to reject claim.

Critical Region Method:

Suppose $\alpha = .05$ for a right-tail test.



$z_{.05}$ "critical z value"

Calculate "test statistic" if the test statistic is in critical region \Rightarrow reject H_0 .
If the test statistic NOT in critical region \Rightarrow fail to reject H_0

Ex! Book 420/9

with $\alpha = .05$ test claim chips ahoy lowfat cookies has less than 24 chips
if a sample of 40 has mean = 19.6 chips
std. dev. = 3.8 chips

claim $\mu < 24$

opp $\mu \geq 24$ not org.

H_0 $\mu = 24$

H_1 $\mu < 24$ left tail \rightarrow

T-test

Stat \rightarrow test



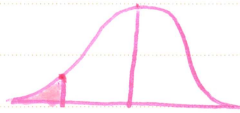
mo 24 T-test \rightarrow Stat

\bar{x} 19.6

SX 3.8

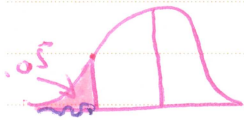
n 40

$\mu < 0$



T-Test: $T = -7.3232$, $P = 0$

P is less than $\alpha \Rightarrow$ reject H_0 not org.
 \Rightarrow data support claim



$$t = \text{invT}(0.05, 39) = -1.68$$

\uparrow
1-tail

critical region -1.68

TRY: With $\alpha = .05$ test claim
mean weight of elephants is less than
2000 lbs if a sample of 20 elephants has
mean weight = 1975 lbs and std dev
for the weight of ALL elephants is 110.

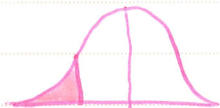
claim: $\mu < 2000$ lbs

opp: $\mu \geq 2000$ not orig.

$H_0: \mu = 2000$

$H_1: \mu < 2000$ left

Calculate z-Test "sample ALL Elephants"
population std. dev.



$$z = -1.206$$

$$p = 0.1146$$

since P-value is not less
than α it fails, H_0 is not
from original. Evidence not
suff. to support claim.

On Final Claims about 2 proportions:

Ex: with $\alpha = .05$ test claim a higher proportion of women have X than men if 262 of 932 women have X
273 of 1082 men have X

$p_1 =$ woman

$p_2 =$ men

Claim: $p_1 > p_2$

opp: $p_1 \leq p_2$

$H_0: p_1 = p_2$ not org.

$H_1: p_1 > p_2$ right tail

STAT \rightarrow tests \rightarrow 2 prop z test

X_1 262

n_1 932

X_2 272

n_2 1082

$p_1 > p_2$



$$z = 1.4717$$

$$P = .0105$$

Draw

P is not less than α , fail to reject H_0
not org.

\Rightarrow not suff.
evidence to
support claim...

Hypothesis Testing cont.

2 proportions

Ex! With $\alpha = .05$ test the claim people support X and Y equally if 262 of 511 support X, 311 of 601 support Y

p_1 support X

p_2 support Y

Claim $p_1 = p_2$

opp $p_1 \neq p_2$

H_0 $p_1 = p_2$ orig

H_1 $p_1 \neq p_2$ 2-tail

2-Prop Z test

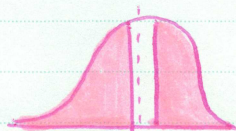
x_1 : 262

n_1 : 511

x_2 : 311

n_2 : 601

$p_1 \neq p_2$



$z = -1.58$ $P = .8745$

P not less than α

\Rightarrow Fail to reject H_0

\Rightarrow Orig. claim

\Rightarrow not suff. evidence
to reject claim

Ex! With $\alpha = .05$, test the claim fewer people have seen a UFO than UFO if 22% of 621 seen UFO and 24% of 816 seen UFO

$$p_1 \text{ UFO } 621 \cdot .22 = 137 \text{ round whole}$$

$$p_2 \text{ UFO } 816 \cdot .24 = 197 \text{ round whole}$$

2 prop-z-test

$$x_1: 137$$

$$n_1: 621$$

$$x_2: 197$$

$$n_2: 816$$

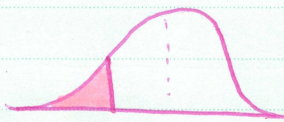
$< p_2$ less than

claim $p_1 < p_2$

opp $p_1 > p_2$

H_0 $p_1 = p_2$ not org.

H_1 $p_1 < p_2$ left



$$z = -1.0344 \quad P = .1505$$

P not less than α

\Rightarrow fail to reject H_0

\Rightarrow not suff. data to support claim...

Book 450/8

Ex! with $\alpha = .01$ test claim ginkgo helps prevent dementia if of 1545 subjects given ginkgo had dementia of 246 developed dementia, had dementia of 1524 subjects given placebo, 277 developed dementia

$p_1 =$ given ginkgo

$p_2 =$ given placebo

claim $p_1 < p_2$

opp $p_1 \geq p_2$

H_0 $p_1 = p_2$ not org

H_1 $p_1 < p_2$ left

Z prop Z-test

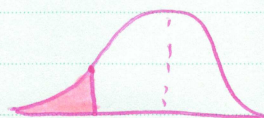
x_1 246

n_1 1545

x_2 277

n_2 1524

$p_1 < p_2$



$z = -1.6601$ $p = .0484$

because $\alpha = .01$ (not the usual)

\Rightarrow Fail to reject H_0

\Rightarrow not org.

\Rightarrow not suff. to support claim...

Draw

TRY: with 2.05 test claim more people have X than Y; if 362 of 521 have X, 185 of 288 have Y

claim $p_1 > p_2$

opp $p_1 < p_2$

H_0 $p_1 = p_2$ (not org)

H_1 $p_1 > p_2$ (right)

Z-prop test - Z

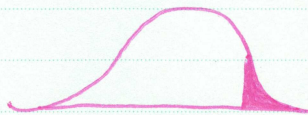
X_1 362

n_1 521

X_2 185

n_2 288

$p_1 > p_2$



$p = .0634$ more

H_0 not org claim

$\Rightarrow p$ is not less than α

\Rightarrow Fail to reject H_0

\Rightarrow There is not suff. evidence to support claim...

The Set-up (with μ as #)

wording?	claim	opp	H_0	H_1
Less than	$p < \mu$	$p \geq \mu$	$p = \mu$ orig.	$p < \mu$ (L)

atmost	$p \leq \mu$	$p > \mu$	$p = \mu$ orig.	$p > \mu$ (R)
--------	--------------	-----------	--------------------	---------------

more than	$p > \mu$	$p \leq \mu$	$p = \mu$ orig.	$p > \mu$ (R)
-----------	-----------	--------------	--------------------	---------------

at least	$p \geq \mu$	$p < \mu$	$p = \mu$ orig.	$p < \mu$ (L)
----------	--------------	-----------	--------------------	---------------

"IS" μ	$p = \mu$	$p \neq \mu$	$p = \mu$ orig.	$p \neq \mu$ 2 tail
------------	-----------	--------------	--------------------	------------------------

7.13.17

Hypothesis testing Cont.

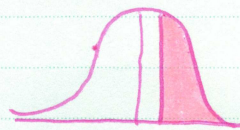
④ 2 independent means

HiW
20

Ex. With $\alpha = .05$ Martians are taller than Venusians if a sample of 34 martians has $\bar{x} = 5.3$, $S = .6$
42 Venusians has $\bar{x} = 5.2$, $S = .8$

 $\mu_1 = \text{Martians}$ $\mu_2 = \text{Venusians}$ Claim: $\mu_1 > \mu_2$ opp. $\mu_1 \leq \mu_2$ H₀: $\mu_1 = \mu_2$ orig.H₁: $\mu_1 > \mu_2$ (R)STAT \rightarrow test \rightarrow 2-Samp T Test $\bar{x}_1: 5.3$ $Sx_1: .6$ $n_1: 34$ $\bar{x}_2: 5.2$ $Sx_2: .8$ $n_2: 42$ $\mu_1: > \mu_2$

↓

Draw $t = -6.223$ $P = .2678$ P not less than α Fail to reject H₀

not original

not suff. data to support...

9.3

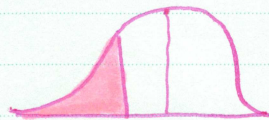
Book 464/8

With $\alpha = .05$ test the claim, men speak less words per-day than women from following data:

\bar{x}	s	n
-----------	---	---

m	15668	8632	186	$\mu_1 \rightarrow$ men
---	-------	------	-----	-------------------------

w	16215	7301	210	$\mu_2 \rightarrow$ women
---	-------	------	-----	---------------------------

Claim: $\mu_1 < \mu_2$ opp: $\mu_1 \geq \mu_2$ Ho: $\mu_1 = \mu_2$ origHi: $\mu_1 < \mu_2$ (L) \bar{x}_1 15668s_{x1} 8632n₁ 186 \bar{x}_2 16215s_{x2} 7301n₂ 210 $\mu_1 < \mu_2$ 

t = .6762

p = .2497

P not less than α

Fail to reject

not from orig. claim

not suff. data to support...

Book 464/11

TRY: With $\alpha = .05$ test claim skulls from 4000 bc have same size as skulls from 150 ad if

n	\bar{x}	s	
30	131	5.1	$\mu_1 = 4000 \text{ bc}$
30	136	5.3	$\mu_2 = 150 \text{ ad}$

Claim: $\mu_1 = \mu_2$

opp: $\mu_1 \neq \mu_2$

$H_0: \mu_1 = \mu_2$ org.

$H_1: \mu_1 \neq \mu_2$ 2-tail

$X_1: 131$ calculator 2-SampTTest

$Sx_1: 5.1$

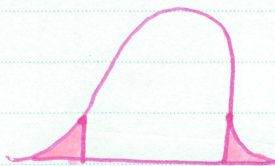
$n_1: 30$

$\bar{x}_2: 136$

$Sx_2: 5.3$

$n_2: 30$

$\mu: \neq$



$$t = 3.7233$$

$$P = 4E-4$$

$$= 4 \times 10^{-4}$$

P is less than α

reject H_0 from original

there is suff. evidence to reject claim...

7-17-17

Hypothesis test Cont.

- ③ Dependent means (match pairs)
- ④ Correlation and regression

Book: Ex 461/1

Hight of the winner

189 173 183 180 179

Hight of the loser

170 185 175 180 179

List of data \Rightarrow matching pairs

with $\alpha = .05$ test claim winner is taller than the loser

$\mu_d =$ mean difference between the first and second list

If the claim is that first list is bigger \Rightarrow claim: $\mu_d > 0$. if the claim is that the second list is bigger \Rightarrow claim: $\mu_d < 0$. if the claim is this list are equal \Rightarrow claim: $\mu_d = 0$

Claim: $\mu_d > 0$

opp: $\mu_d \leq 0$

H₀: $\mu_d = 0$ *orig*

H₁: $\mu_d > 0$ (2)

① STAT \rightarrow ClrList L1, L2, L3 end

② L1 enter first list \leftarrow
L2 enter second list \leftarrow

STAT \rightarrow Edit \rightarrow enter

L1	L2
189	170
173	185
183	175
180	180
179	178

2nd Quit (get out)

③ from home screen

L1 - L2 \rightarrow L3 enter

\uparrow
minus

\uparrow
1sto

It shows: $\{ 19 -12 8 \emptyset 13$

T-test \Rightarrow Data: STAT \rightarrow Test \rightarrow T-test
Data \leftarrow

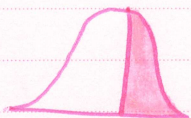
$\mu_0: 0$

List: L3

Freq: 1

$\mu: > \mu_0$

Draw



$t = .6283$ $P = .2819$

P not less than α

Fail to reject H_0

Not org.

Not suff. data to support claim...

Ex: With $\alpha = .05$ test claim blood pressure is the same in right and left arm from following data.

Right: 120 118 127 138 140 110

Left: 117 119 122 145 142 115

Claim: $\mu_d = 0$

opp: $\mu_d \neq 0$

$H_0: \mu_d = 0$ from org

$H_1: \mu_d \neq 0$ 2-tail

① STAT → Clr List L_1, L_2, L_3 2nd

② STAT → Edit → enter

L_1	L_2
120	117
118	119
127	122
138	145
140	142
110	115

2nd Quit

③ from home screen

$L_1, L_2 \rightarrow L_3$ Enter

↓

sto

it shows: $\{3 -15 -7 -2 -5\}$

STAT → Test →

↓

T-test → Data

μ_0 : 0

List: L_3

Freq: 1

μ : \neq

Draw



$\hat{\mu} = .6241$ $P = .5599$

P not less than α ,

Fail to reject H_0 ,

from org. μ

Not suff data to,

reject

Claim...

Ex: With $\alpha = 0.05$ test claim that
Fri 13 is unlucky from following
accident data

Fri 6: 8 10 7 11 8 7

Fri 13: 10 12 5 11 9 11

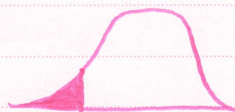
Claim: $\mu_d < 0$

opp: $\mu_d \geq 0$

H_0 : $\mu_d = 0$ ~~or~~

H_1 : $\mu_d < 0$ (L)

Calculator \rightarrow



$t = -1.4$ $P = .1102$

Fail to reject

not org

not suff. data to support claim...

P is not less than α

TRY: with $\alpha = .05$, test claim
spouse 1 speaks more words than
spouse 2

Spouse 1: 68 71 62 111

Spouse 2: 60 81 22 106

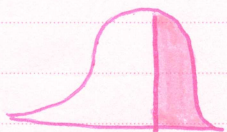
Claim: $\mu_d > 0$

opp: $\mu_d \leq 0$

H_0 : $\mu_d = 0$ orig

H_1 : $\mu_d > 0$ (R)

$\{ 8 \ -10 \ 11 \ -2 \ 5 \}$ calculator [↑]



$t = .6356$ $P = .2798$

more

P is not less than α

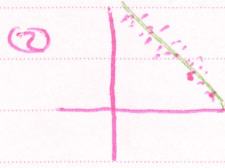
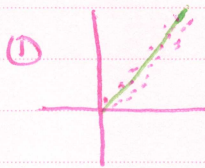
H_0 not orig

Fail to reject H_0

There is not suff evidence to support H_1 .



Correlation and Regression



Sample points from a population
from data points. (ordered pair x, y)

① These data points are linearly correlated to a straight line w/
positive slope r is close to 1

↓

"Correlation coefficient"

③ These data points don't correlate
to any straight line r is close to 0

② These data points are linearly
correlated to straight line with
negative slope r is close to -1

Where we are headed!

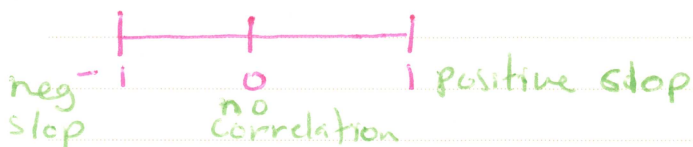
Give a set of sample data points:

- ① Do these points come from a linearly correlated population?
- ② If so, find the equation of the regression line.

7-18-17

Correlation and Regression cont.

r = sample correlation coefficient

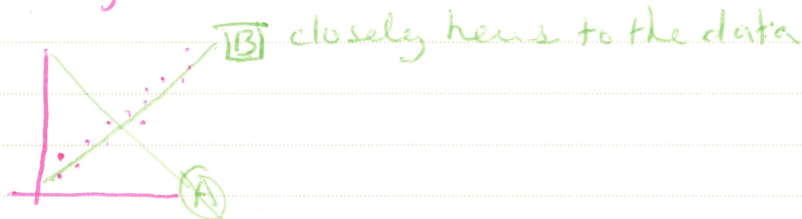


Ex! With $\alpha = .05$ test the claim, the following data from a linearly correlation population find r , the regression equation, use the regression equation to predict y when $X = 6$, plot the data and graph the regression line.

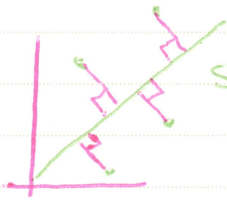
$X: 1 \ 2 \ 3 \ 4 \ 5$

$y: 6 \ 7 \ 8 \ 9 \ 10$

regression line



The regression line is the unequally determined line that minimizes the data points to the line



sum of distance data point

Sample

population

$\frac{1}{n}$
 \bar{x}

μ

mean

s

σ

std. dev.

r

ρ (rho)

Correlation

Claim: $\rho \neq 0$ (not "0")

opp: $\rho = 0$

H_0 : $\rho = 0$

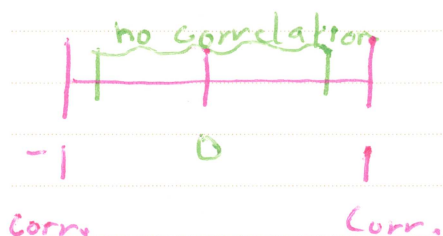
H_1 : $\rho \neq 0$

ρ close to 1 \Rightarrow correlation to a line w/ pos. slope

ρ close to -1 \Rightarrow correlation to a line with neg. slope.

ρ close to 0 \Rightarrow no correlation

If there is correlation P close to 1 or -1, but not close to 0



On calculator:

① STAT
↓
Clr list L_1, L_2

② STAT → edit
* X-value → $L_1: 12345$
Y-value → $L_2: \cancel{6789}10$
 257910

③ STAT → Tests
↓
F: Lin Reg TTest

Xlist: L_1 Vars → Y-vars
Ylist: L_2
Freq: 1
P: \neq
RegEQ: Y_1 ↗

Function
 $Y_1 \leftarrow Y_1$
Eantet

$$P = .002128 \dots$$

$$\dots 45.6$$

$$b = 2$$

$$s = .6324$$

$$r^2 = .9707$$

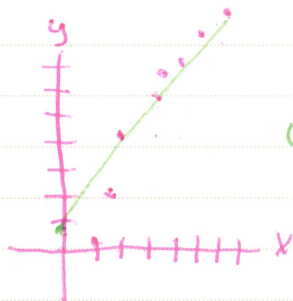
$$r = .9853$$

Test claim: $P = .002$ "Does ... support claim"
 P less than α
reject H_0
not org.
data support claim

Find r : .9853292782 copy ALL

regression equation: $y = a + bx$
 $= .6 + 2x$

when $x = 6$: $y = .6 + 2(6) = \boxed{12.6}$



Graph and plot points
on calculator

turn on
 $\boxed{\text{Plot 1}} \rightarrow \text{enter}$
 $\text{Y} =$ key $y = .6 + 2x$ (turn off...)
2nd $\boxed{\text{stat plot}} \rightarrow \text{enter}$

On OFF

type   

Xlist: L1

Ylist: L2

mark:  +.

Turn plot 1 on (arrow \uparrow to it and enter as a toggle to turn it "on/off")

Window Key

X_{min} = 1 less than smallest
x-value in data

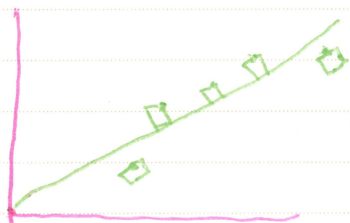
X_{max} = 1 more than largest x-value

X_{sc} = 1

Y_{min} = 1 less than smallest
y-value

Y_{max} = 1 more than largest y-value

Graph Key



$$\left(\begin{array}{c} \text{for } \alpha = .05 \\ \hline -1 \quad | \quad .818 \quad | \quad .818 \quad | \quad 1 \end{array} \right)$$

ex: with $\alpha = .05$

- ① Test claim the data are from a linearly correlated population
- ② find r
- ③ find regression equation
- ④ Use regression to predict y when $X=12$
- ⑤ graph points and equation

$$X: 2 \quad 4 \quad 6 \quad 8 \quad 10$$

$$y: 14 \quad 15 \quad 10 \quad 6 \quad 3$$

claim: $\rho \neq 0$

opp: $\rho = 0$

H_0 : $\rho = 0$ (not corr.)

H_1 : $\rho \neq 0$ (2-tail)

Data input

Lin Reg TT test

$$P = 2.22 \times 10^{-4}$$
$$= .000222$$

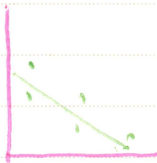
$$R = -.9967441086$$

$$y = 22.9 - 2.05X$$

P is less than α
reject H_0 , not org., data support claim..

$$\text{when } X=12 \quad y = 22.9 - 2.05(12) = -1.7$$

Graph: 1, 11, 2, 20 (??)



Ex: $X: 2 \ 4 \ 6 \ 8 \ 10$
 $y: 1 \ 2 \ 11 \ 5 \ 9$

Claim: $P \neq 0$

Opp: $P = 0$

$H_0: P = 0$ not org.

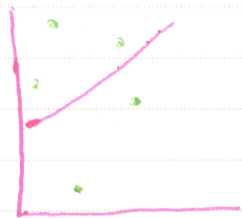
$H_1: P \neq 0$ 2-tail

Calculator . . .

① $P = .603$, not less than α
Fail to reject H_0
not org. claim
not suff. data to support claim

② $r = .3168751111$

③ $y = a + bx$
 $y = 4.7 + .35x$



④ $y = 4.7 + .35(12) = 8.9$

⑤ Graph: $X_{\min} 1$ $Y_{\min} 1$
 $X_{\max} 11$ $Y_{\max} 12$
 $X_{cc} 1$

Regression Practice

①

X	1	2	3	4	5	6	$x=6$
Y	11	9	6	4	2	?	

claim: $\rho \neq 0$

opp: $\rho = 0$

H_0 : $\rho = 0$ not org

H_1 : $\rho \neq 0$ 2-tail

$$p = 1.8010^{-4} = .00018$$

$$r = .997176465$$

$$y = 13.3 + (-2.3)x$$

$\alpha = 13.3$
 $\beta = -2.3$

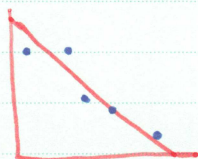
$$13.3 - 2.3(6) = -1.5$$

when $x=6$ $y = -1.5$

p is less than α , reject H_0

H_0 not org claim

The sample data support claim...



②	X	2	4	6	8	10	12	$x = 12$
	y	3	7	9	14	19	?	

claim: $p \neq 0$

opp: $p = 0$

H_0 : $p = 0$ not org

H_1 : $p \neq 0$ 2-tail

$$p = .001$$

$$r = .989962511$$

$$a = 1.3$$

$$b = 1.95$$

$$y = 1.3 + 1.95(12) = 24.7$$

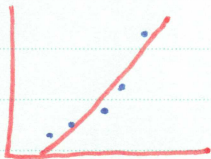
$$\alpha = .05$$

when $x = 12$ $y = 24.7$

p is less than org. claim

Reject H_0 , H_0 not from org. claim

sample data supports claim...



③ X 1 2 3 4 5 6
 Y 7 3 5 1 4 ?

claim: $P \neq 0$

opp: $P = 0$

$H_0: P = 0$ not org.

$H_1: P \neq 0$ 2-tail

$$P = .320$$

$$r = -.5656854249$$

$$a = 6.4$$

$$b = -.8$$

$$y = 6.4 - .8(6) = 1.6$$

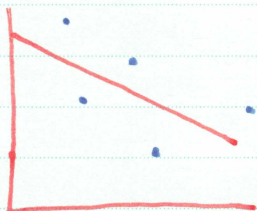
$$\alpha = .05$$

when X is 6 $y = 1.6$

P is not less than α

fail to reject H_0 , not org.

Not suff. data to support claim...



Review: Hypothesis Test

- ① 2 proportion
- ② 2 independent means
- ③ 2 dependent means
- ④ Correlation - regression

Hyp. test
Regression/Corr.
T-Test
Samp. / Pop. mean

- ① Ex: with $\alpha = .05$, test the claim the Maple Leafs challenges have a higher proportion of success than Sharks Challenges if maple Leafs were successful 62 of 173
Sharks successful 41 of 125

P_1 Leafs
 P_2 Sharks

Claim $P_1 > P_2$

opp $P_1 \leq P_2$

$H_0 P_1 = P_2$ OK

$H_1 P_1 > P_2$ (R)

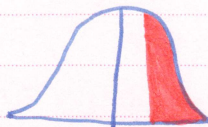
2 Prop - Z test

X_1 62

N_1 173

X_2 41

N_2 125



$Z = .5442$

$P = .2932$

Fail to reject, not from org.

Not suff. data to support claim.

② Ex: with $\alpha = .05$, test claim that cats are lighter than dogs if a

\bar{x} s n

Cats 12.6 1.4 37

Dogs 14.3 3.2 33

$\mu_1 = \text{Cats}$ $\mu_2 = \text{Dogs}$

claim $\mu_1 < \mu_2$

opp $\mu_1 \geq \mu_2$

H_0 $\mu_1 = \mu_2$ orig

H_1 $\mu_1 < \mu_2$ \odot

2 Samp TTtest

\bar{x}_1 12.6

s_{x1} 1.4

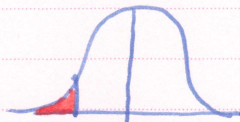
n_1 37

\bar{x}_2 14.3

s_{x2} 3.2

n_2 33

$\mu <$



$t = 2.8205$

$P = .0036$

reject H_0 , not orig.
Data support claim...

③ Ex: With $\alpha = .05$, test claim when aliens from mars meets with aliens from Vulcan. Their heights are the same.

Mars 14 16 18 13 12 } STAT
Vulcan 15 14 20 10 15 } list

claim $\mu_d = 0$

opp $\mu_d \neq 0$

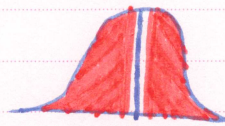
H_0 $\mu_d = 0$ org

$\mu_1 \mu_2 \neq 0$ z-test

$L_1 - L_2 \rightarrow L_3$ (-1 2 -2 3 -3)

T-Test (data)

$\mu \neq 0$



$$t = -.1728$$

$$p = .8712$$

Fail to reject H_0 , from org.
not suff. data to reject claim

④ Regression & Correlation

with $\alpha = .05$ test claim these data are from a linearly correlated population find r , reg. eq., predict y when $x = 30$, graph all.

X	5	10	15	20	25	} stat list
y	120	130	137	142	153	

$$x = 30 \quad \text{Claim } p \neq 0$$

$$\text{opp } p = 0$$

$$H_0: p = 0 \text{ ok}$$

$$H_1: p \neq 0 \text{ 2-tail}$$

LinReg TTest

$$P = 7.25 \times 10^{-4} = .000725$$

reject H_0 , not org., data support claim...

$$r = .9928454366$$

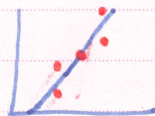
$$y = 113 + 1.56x$$

$$113 + 1.56(30) = 159.8$$

Y = Calculate

X min ... Y min ...

X max ... Y max ...



Practica Test

① Believe in ghosts/aliens

P_1 Believe in ghosts

P_2 Believe in aliens

$$\alpha = .05$$

216 of 1377

255 of 1711

more than
"higher"

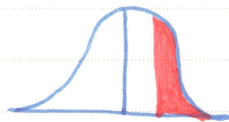
Claim: $p_1 > p_2$

opp $p_1 \leq p_2$

H_0 $p_1 = p_2$ org

H_1 $p_1 > p_2$ Ⓒ

2 prop z-test



$$z = .6013 \quad p = .2738$$

P is not less than α , H_0 not from org.
Fail to reject, there is not suff.
evidance to support claim

② Raccoons v.s. Porcupines

μ_1 raccoons

μ_2 porcupines

2 samp TTest

$\alpha = .05$

61 raccoons

\bar{x} 2.3 ft

S .3 ft

55 porc.

\bar{x} 2.23 ft

S .31 ft

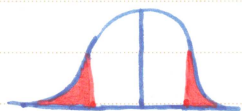
claim $\mu_1 = \mu_2$

opp $\mu_1 \neq \mu_2$

H_0 $\mu_1 = \mu_2$ org

H_1 $\mu_1 \neq \mu_2$ 2-tail

Equal



$t = 1.2331$

$P = .2201$

P is not less than

H_0 from org., Fail to reject H_0

There is not suff. evd. to reject claim...

③ Fewer words

stat List

Spouse 1 458 320 567 444 433 485
Spouse 2 455 301 508 492 403 513

claim $\mu < 0$

opp $\mu \geq 0$

H_0 $\mu = 0$ o/s

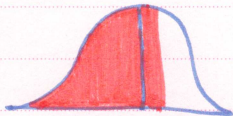
H_1 $\mu < 0$ (L)

$\alpha = .05$

less than

T-Test "Data"

(319 59 -48 30 -28)



$t = .3657$ $P = .6352$

P is not less than α

H_0 is not from org. claim

Fail to reject H_0

There is not suff. evidence
to support claim...

④ Blood Pressure

stat list

Right: 130 120 118 122 109 123

Left: 125 109 119 120 110 115

claim: $\mu_d > 0$

opp: $\mu_d \leq 0$

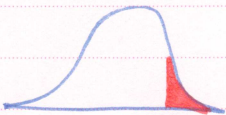
$H_0: \mu_d = 0$ oky

$H_1: \mu_d > 0$ (a)

$< .05$
less than

T-Test "Data"

(5 11 -1 2 -1 8)



$t=2$

$P=.051$

P is not less than α
fail to reject H_0 ,
 H_0 not from org. claims
There is not suff. evidence
to support claim

⑤ Correlation / Regression

X	1	2	3	4	5	6	X=6, $\alpha=0.05$
y	9	7	6	4	1	?	

Claim $\rho \neq 0$

opp $\rho = 0$

$H_0: \rho = 0$ ok

$H_1: \rho \neq 0$ 2-tail

$$p = .002$$

$$r = .98510411$$

$$a = 11.1$$

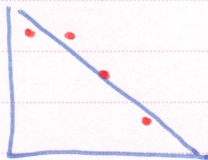
$$b = -1.9$$

$$y = 11.1 - 1.9(6) = -.3$$

when X is 6 $y = -.3$

P is not less than α
Fail to reject H_0 ,
Not significant

There's not suff. evid. to support...



6

X	2	4	6	8	10	12	X=12
Y	15	7	10	12	1	?	$\alpha = .05$

claim $P \neq 0$

opp $P = 0$

H_0 $P = 0$ not org

H_1 $P \neq 0$ 2-tail

$$P = .205$$

$$r = .6812012094$$

$$y = 15.9 - 1.15(12) = 2.1$$

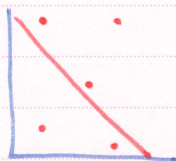
when $X = 12$ $Y = 2.1$

P is not less than 2

Fail to reject H_0

H_0 not from org. claim

There's not suff. evd. to support...



⑦

\bar{X} = Sample mean
 μ = population mean

When a word problem asks for a sample, then we use \bar{X} - such as a drug sample. When a word problem asks for ALL or population is used we use the μ symbol.

Ex: Constructs 98% CI containing the mean # of choco. chips in a cookie. a sample of 33 cookies has $\bar{X} = 20.6$ and population std. dev. for all cookies is 5.3.

Ex: With $\alpha = .05$ test the claim mean ft. of giraffes has mean ft = 17.55 ft and std dev. = 1.3 ft